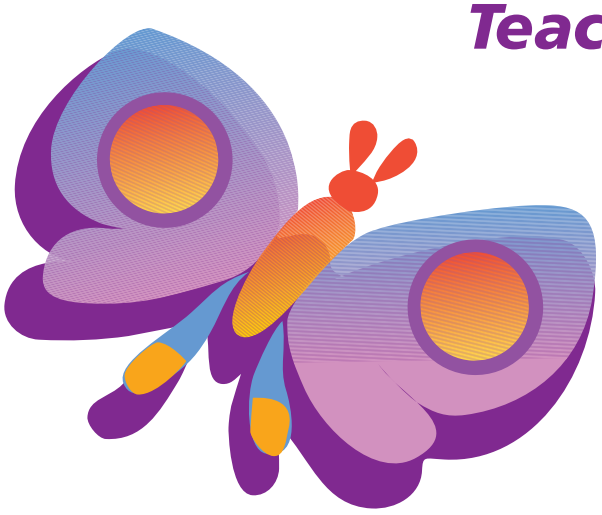
Counting On

Teaching activities



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DEPARTMENT
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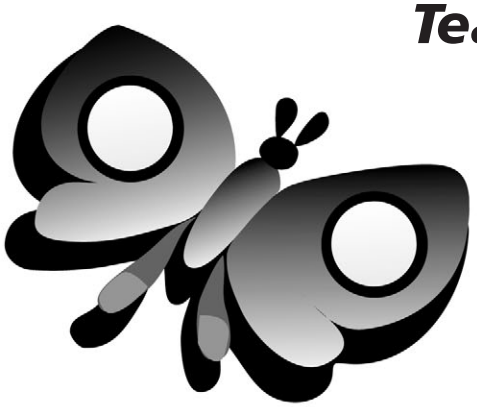

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Contents

Teaching a *Counting On* lesson

The role of mental computation

Many of us may recall a focus on mental arithmetic in our own schooling. This tended to be aligned to learning and recalling number facts and multiplication tables. What was important was speed and accuracy. The *human calculator* took in numbers and produced answers quickly. With the development of cheap hand-held calculating devices the role of mental arithmetic diminished in schools.

During the past ten years, research has started to recognise the value of *mental computation* in developing a rich understanding of place value and our number system. Mental computation is used in a broader sense than traditional mental arithmetic. While not neglecting the correctness of the answer, mental computation emphasises the mental processes used to achieve the answer.

From the research we know that:

- *children may invent their own strategies for calculating mentally* (Kamii et al, 1993)
- *mental strategies frequently differ from written methods. For example, many mental strategies are "front end", starting from the left whereas written methods for addition, subtraction and multiplication start from the right* (Askew, 1997).
- *some students replicate written methods as mental strategies* (Perry & Howard, 2000)
- *methods vary from child to child and even the same child may choose to use different methods to solve similar problems at different times* (Hope & Sherrill, 1987)
- *some mental strategies are more efficient than others* (Denvir & Brown, 1986, Steffe & Cobb, 1984).

To develop mental computation in the classroom, teachers can encourage students to explain how they arrived at their answers and to compare their strategies with those of other students.

Working in the zone

Lessons within the *Counting On* project have several unique characteristics. Teaching is informed by an initial comprehensive interview and on-going assessment as part of the teaching process. Consequently, teaching intentionally builds on the most advanced strategies which students are currently using independently.

The teaching approach is problem-based. Students are routinely engaged in thinking hard to solve arithmetical problems which, for them, are quite challenging. The teaching is focused just beyond the leading edge of students' knowledge.

The teaching is designed to develop a conceptual understanding of place value to support mental computation and the structure of groups.

Explicit teaching

Students in the *Counting On* project often have a deeply entrenched preference for methods of counting by ones. As Gray states, "… by the end of the middle years of schooling such children feel secure, often confident, in their procedure. It is successful, may well have been refined and it leads to solutions. Why change? What need is there to look for alternatives?" (1991, p. 570).

Teachers attempting to support students in the transition to group-based solution methods are unlikely to succeed unless their students understand the need for this transition. Explicit teaching of students in *Counting On* enables this partnership in learning to develop. Sharing the responsibility for learning and the intent of the classroom activities in *Counting On* also enables students to realise the progress that they are making. The teaching explicitly assists the movement of students from unitary to composite-based mental strategies.

Developing knowledge of number blends to ten is a start but there is also a need to develop a part-whole knowledge of numbers to twenty. This makes effective use of the blends to ten combined with knowledge of the ten in numbers.

Teaching activities in *Counting On* often make use of novel approaches to avoid in-built resistance by students and negative attitudes that can surface by retracing old paths.

Teaching sequences

To develop a robust understanding of place value and our base ten system of numbers, students need to develop a range of skills. The capacity to flexibly separate and recombine numbers is an integral component of almost all number work. Consequently, *combining* and *partitioning* will generally form part of a teaching sequence in *Counting On*.

As well as addressing specific processes and concepts, teaching sequences can be arranged according to the specific context or model being used. That is, it is possible to identify the specific settings within which different concepts may be developed. For example, word problems, finger patterns, ten-frames, the empty number line, the hundred chart or the Chinese abacus all provide different settings to develop place value concepts. Some settings will be better suited to some concepts.

The teaching activities that follow identify both the concepts being addressed as well as the context or model being used. Exploring one or two models in depth is preferable to using many different models for the same idea. The linear model, progressing from the clothesline to the empty number line and on to the number ribbon also provides a natural sequence within the model.

Stage 3

The Stage 3 logo is used to indicate activities that may be more suited for introducing concepts to Stage 3 students.

Additional activities will be available through the web site. This can be accessed through **www.schools.nsw.edu.au/learning/k_6/maths/index.php**

References

Askew, M. (1997). Mental methods of computation. *Mathematics Teaching*, 160, 7–8.

Carraher, T. N. (1988). Street mathematics and school mathematics. In A. Borbas (Ed) *Proceedings of the Twelfth International Conference for the Psychology of Mathematics Education* (Vol. 1, pp. 1–23). Veszprém (Hungary): International Group for the Psychology of Mathematics Education.

Denvir, B. & Brown, M. (1986a). Understanding of number concepts in low attaining 7–9 year olds: Part 1. Development of descriptive framework and diagnostic instrument. *Educational Studies in Mathematics*, 17, 15–36.

Denvir, B. & Brown, M. (1986b). Understanding of number concepts in low attaining 7–9 year olds: Part II. The teaching studies. *Educational Studies in Mathematics*, 17, 143-164.

Hope, J. A. & Sherrill, J. M. (1987). Characteristics of unskilled and skilled mental calculators. *Journal for Research in Mathematics Education*, 18 (2), 98–111.

Gray, E. M. (1991). An analysis of diverging approaches to simple arithmetic: Preference and its consequences. *Educational Studies in Mathematics* 22, 551–574.

Kamii, C., Lewis, B. A. & Livingston, S. J. (1993). Primary arithmetic: children inventing their own procedures. *Arithmetic Teacher*, 41 (4), 200–203.

Moss, J. & Case, R. (1999). Developing Children's Understanding of Rational Numbers: A New Model and an Experimental Curriculum. *Journal for Research in Mathematics Education*, 30 (2), 122–147.

Perry R. & Howard P., (2000). Evaluation of the impact of the *Counting On* program. NSW Department of Education and Training.

Steffe, L. P. & Cobb, P. (with E. von Glasersfeld). (1988). *Construction of arithmetic meanings and strategies*. New York: Springer-Verlag.

Some *Counting On* syllabus links from Mathematics K–6

NS2.1 Counts, orders, reads and records numbers up to four digits

The student can, for example

- read and write numbers up to 9999 in numerals and words
- count forwards and backwards by tens or hundreds, on and off the decade
- order numbers up to 9999
- identify the place value in a three-digit number
- use standard and non-standard partitioning of two-digit numbers
- place a set of four-digit numbers in ascending or descending order.

NS3.1 Orders, reads and writes numbers of any size

The student can, for example

- read, write and order numbers of any size using place value
- record numbers in expanded notation
- recognise the location of negative numbers in relation to zero
- use standard and non-standard decomposition of three-digit numbers
- draw a number line and position whole numbers on the line.

NS2.2 Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

The student can, for example

- use part-whole knowledge of numbers to "bridge" to the next decade
- increment by tens and ones from the middle of a decade: jump method
- collect tens from within numbers: split method
- use an empty number line to record mental strategies for addition and subtraction of two-digit calculations
- use complementary facts to solve addition and subtraction problems
- use the hundreds chart to demonstrate solutions to addition and subtraction tasks

NS3.2 Selects and applies appropriate strategies for addition and subtraction with counting numbers of any size

The student can, for example

- mentally add or subtract numbers to 1000
- flexibly use the jump, split or compensatory method according to the demands of the task
- use an empty number line to record mental strategies for addition and subtraction of two- and three- digit calculations

NS2.3 Uses mental and informal written strategies for multiplication and division

The student can, for example

- recall number facts up to 10 x 10
- find multiples and squares of numbers
- use mental and informal written strategies for multiplying or dividing a two-digit number by a one-digit operator
- interpret remainders in division problems
- determine factors for a given number

NS3.3 Selects and applies appropriate strategies for multiplication and division

The student can, for example

- coordinate composite units in dealing with multiplication and division tasks
- select and apply appropriate mental, written or calculator strategies for multiplication and division

WM2.2 Selects and uses appropriate mental and written strategies, or technology, to solve problems

WM2.3 Uses appropriate terminology to describe, and symbols to represent, mathematical ideas

WM2.4 Checks the accuracy of a statement and explains the reasoning used

WM3.2 Selects and applies appropriate problem-solving strategies, including technological applications, in undertaking investigations

WM3.3 Describes and represents a mathematical situation in a variety of ways using mathematical terminology and some conventions

WM3.4 Gives a valid reason for supporting one possible solution over another

The student can, for example

- estimate reasonable answers to tasks involving whole numbers
- access a range of strategies to solve problems involving whole numbers
- modify strategies to meet the need of a particular task
- estimate reasonable answers to computation tasks
- discuss suitable methods for checking answers, e.g. associated additions, commonsense answers, use of calculator.

Some *Counting On* syllabus links from Mathematics 7–8

N4.1 Recognises the properties of special groups of whole numbers and applies a range of strategies to aid computation

The student can, for example

- read, write, interpret and order numbers of any size
- recognise and state place value. The understanding of place value is the most important mathematical concept for Counting On students. It must be emphasized, reinforced and continually revisited.
- read and write in expanded form, e.g. 2536 = 2 x 1000 + 5 x 100 + 3 x 10 + 6 x 1
- state that 2536 can also be described as 25 hundreds and 36 ones or 253 tens and 6 ones.
- draw a number line and position whole numbers on the line
- give automatic responses to single-digit addition and subtraction facts (combinations)
- flexibly use the jump, split or compensatory method according to the demands of the task
- use an empty number line to record mental strategies for addition and subtraction of two- and three- digit calculations
- list multiples of a given number
- instantly recall multiplication facts up to 10 x 10
- estimate results being alert to unreasonable answers.
- apply a range of mental strategies to aid computation, e.g. 3 x 98 is 3 lots of 100 less 3 lots of 2. That is 300 – 6. Why?
- apply simple tests of divisibility mentally as an aid to calculation e.g. if the digits of a number add up to 3, 6 or 9, that number is divisible by 3. Students should be encouraged to create patterns on a hundreds square and on Pascall's triangle to discover simple tests of divisibility.
- distinguish between odd and even numbers
- recognise and represent figurate numbers (triangular, square, etc.) palindromic and fibonacci numbers; tetrahedral, cubic, etc.

WMS4.2 Analyses a mathematical or real-life situation, solving problems using technology where appropriate

WMS4.3 Uses mathematical terminology and notation, algebraic symbols, diagrams, text and tables to communicate mathematical ideas

WMS4.4 Identifies relationships and the strengths and weaknesses of different strategies and solutions, giving reasons

The student, for example

- verifies the use of an efficient strategy to solve mathematical problems
- discusses when it is appropriate to use a mental or written process or a calculator to solve a particular task
- is able to interpret a calculator display in formulating a solution to a problem
- solves a variety of real-life problems involving fractions, decimals and percentages

Activities to support levels of conceptual development in place value

Base ten and place value strategies

Level	What students can do	Students are moving to
Level O Ten as a count	• Use single units in counting. • See 10 as composed of 10 ones **or** one 10, **not both**. • Reconstruct units of 10 by counting them. • Know multiples of 10 as a sequenced count.	• Treat 10 as a single unit **and** recognise it as 10 ones. • Use the structure of 10 as a countable unit. • Use tens and ones in standard partitioning, e.g. 26 = 20 + 6. • Increment by tens and ones from the middle of a decade. • Know place value for 2-digit numbers.
Level 1 Ten as a unit	• Treat 10 as a single unit **and** recognise it as 10 ones. • Use the structure of 10 as a countable unit. • Use tens and ones in standard partitioning, e.g. 26 = 20 + 6. • Increment by tens & ones from the middle of a decade. • Know place value for 2-digit numbers.	• Treat 10 as a unit that can be repeatedly constructed (an iterable unit) e.g. 7, 17, 27, 37. • Use non-standard partitioning of 2-digit numbers, e.g. 74 = 60+14. • Collect ten from within numbers (abstract collectible unit) 37 + 2 4 = 50 + 10 + 1.
Level 2 Tens & ones **Level 2a** Jump method	• Treat 10 as a unit that can be repeatedly constructed (an iterable unit), e.g. 7, 17, 27, 37. • Hold onto one number and count on by tens and ones, e.g. 37 + 25 = 37 + 20 + 5 so 37, 47, 57 and 5 is 62.	• Use 100s, 10s and ones in standard partitioning 234 = 2 x 100s, 3 x 10s & 4 ones. • Treat 100 as10 groups of 10 • Increment by 100s, 10s and ones by holding one number, e.g. 354 + 236 354 + 200 = 554 554 + 30 = 584 584 + 6 = 590
Level 2b Split method	• Separate 2-digit numbers into tens and ones and then regroup, 37 + 25 = 30 + 7 + 20 + 5 = 50 + 12 **or** 50 + 10 + 2 = 62. • Collect ten from within numbers (abstract collectible unit) 37 + 25 = 50 + 10 + 2. • Use non-standard partitioning of 2-digit numbers 74 = 60 + 14.	• Use 100s, 10s and ones in standard partitioning 234 = 2 x 100s, 3 x 10s and 4 ones. • Flexibly treat 100 as 10 groups of 10. • Increment by 100s, 10s and ones by holding one number, e.g. 354 + 236 300 + 200 = 500 50 + 30 = 80 4 + 6 = 10 500 + 80 + 10 = 590
Level 3 Hundreds tens and ones **Level 3a** Jump method	• Use 100s, 10s and ones in standard partitioning, 234 = 2 x 100s, 3 x 10s and 4 ones. • Flexibly treat 100 as 10 groups of 10. • Increment by 100s, 10s and ones by holding one number, e.g. 354 + 236 354 + 200 = 554 554 + 30 = 584 584 + 6 = 590	• Mentally add and subtract numbers to 1000. • Approach numerical problems flexibly, e.g. 368 + 143 360 + 140 = 500 500 + 8 + 3 = 511. • Use non-standard partitioning with 3-digit numbers, e.g. 376 = 200 + 170 + 6.

Level	What students can do	Students are moving to
Level 3b Split method	• Mentally add and subtract numbers to 1000. • Approach numerical problems flexibly, e.g. treat 368 + 143 as 360 + 140 = 500 and 8 + 3 = 11 so 511. • Use non-standard partitioning with 3-digit numbers, e.g. 376 = 200 + 170 + 6.	• Use 10ths and 100ths to represent fractions. • Understand the positional place value of decimals. • Know 10 increments of 1 tenth equals and increment of 1. • Count by tenths using ones and tenths • Mentally calculate using tenths by bridging to one
Level 4 Decimal place value	• Compare the size of decimal fractions. • Understand the positional place value of ragged decimals. • Know 10 increments of 1 tenth equals an increment of 1. • Count by tenths using ones and tenths. • Mentally calculate using tenths by bridging to one.	• Understand that place value (as powers of ten) can extend in two directions. • Explain calculations involving thousands and thousandths. • Understand that the place value of a digit increases by multiples of 10 as it moves to the left and decreases by divisors of 10 as it moves to the right.
Level 5 System place value	• Understand that place value (as powers of ten) can extend indefinitely in two directions; left and right of the decimal point. • Explain calculations involving thousands and thousandths. • Understand that the place value of a digit increases by multiples of 10 as it moves to the left and decreases by divisors of 10 as it moves to the right.	• Apply understandings of place value to a range of representational systems, e.g. repeating decimals.

Warm-up activities

1. Guess my number

Think of a number between 1 and 100 and challenge students to ask questions that will identify the number in the least number of guesses. The person who has selected the number is only allowed to answer *yes* or *no*. Good questions that focus on cutting out extraneous numbers can be identified such as "is it more than 50?" or "is it an even number?"

The range of the numbers being considered can be extended to three digits and decimals.

Variation

Use sticky labels to place a number on each student's back. Students move around and may ask one question of each person they meet. That person can only answer *yes* or *no*. Students sit down when they have identified their number.

2. How did you do it?

Put single-digit addition questions on the board in a horizontal form and ask students to share how they solved the problem, e.g. 7 + 5

Possible responses might be 7 + 5 = 7 + 3 + 2 = 10 + 2 = 12, 6 + 6 = 12 so 7 + 5 = 12.

Variation

Extend the examples to include larger numbers or other operations, e.g. 17 + 4, 37 + 5, 49 + 4, 5 x 99.

3. Activities using the numeral cards 0–100.

- Hand out a sequence of the cards to match number of students in the class. Students, in silence, place themselves in ascending order. Placing the numbers in order can be used to reinforce forward and backward counting or odd and even numbers. By selecting appropriate numeral cards, students can practise counting by multiples and play the game of multiple buzz where students turn the number face up if it is a selected multiple.
- Hand out a sequence of cards to match number of students in the class. Invite students to place the numeral cards on the floor to form the first row of the hundreds chart. When the first 30 numbers are in place, the next 70 are distributed. Have students construct the hundreds chart by following patterns such as counting by 10s beginning with 7, completing the row for the 50s, adding multiples of five. Allow time to discuss patterns.
- Reconstruct a calendar month. Hand out the numeral cards 1–30 or 1–31. Make and distribute additional cards for the days of the week. Indicate that you need to construct a calendar for a certain month and all you know is that Saturday is the 8th. Students work together to reconstruct the calendar beginning with the cards for Saturday and 8. When completed, discussion could focus on vertical patterns such as adding 7, diagonal patterns of 6 and 8, and adding the numbers in a 2 x 2 square.

- *Maths Tipping* is an activity where students spread out and stand around the classroom. The teacher displays one or more cards and asks a question such as double this number, multiply this number by 5, add these two numbers. The student who answers correctly tips another student who sits down and is out of the game. The last student left standing is the winner.
- *Team ordered numbers*. Arrange students in two teams and distribute chosen numeral cards randomly. The teams compete against each other with the first team to order the numbers correctly being the winner. For example a set of cards may be 66611, 66161, 66116, 61661, 61616, 61166.
- *100 concentration* is an activity for pairs of students. Using the numeral cards 0, 10, 20, 30, 40, 50, 60, 70, 80, 90 and 100 students take turns to turn over two cards. If they add to 100 they keep the cards. This activity can be used with any sequence of cards. For example the 0–10 cards can be used to practise combinations to 10 and 0–20 to practise combinations to 20. This game can also be played as the game, *Fish*.
- *First to 21* is a game for 2 players. The numeral cards 1–21 are set out in sequence. The first player starts at 0, adds 1, 2 or 3 to 0 and turns the card they land on face down. The second player adds 1, 2 or 3 to the first player's total. The players take turns until one player reaches 21 and is the winner. This is a strategy game and discussion should focus on what strategies may help to win the game.

4. The answer is... what is the question?

Write a number on the board such as 25. Allow time for students to generate questions to match the answer. Share the answers and challenge students to vary their responses, e.g. 20 + 5, 10 + 10 + 5, 100 - 75, 4 x 5 + 5.

Variation

Record the responses and catergorise them. For example addition, subtraction, multiplication, division or fraction.

5. Work it out

Write an algorithm on the board such as 47 + 39 and allow students time to mentally calculate the answer. Invite students to share their strategies and record the methods beside the student's name.

Scott's method 47 + 30 = 77, 77 + 9 = 86
Kelly's method 40 + 30 = 70, 7 + 9 = 16, 70 + 16 = 86
Tran's method 47 + 40 = 87, 87 - 1 = 86
Josh's method 50 + 40 = 90, 90 - 4 = 86.

Write another algorithm and ask students to choose a different strategy to the one they used the first time.

6. Looking for patterns

Write an algorithm on the board and ask the students to work with a partner to record related facts.
Example: 67 + 25 = 92
Related facts: 57 + 35 = 92, 65 + 27 = 92, 92 - 65 = 27.

Unit 1: Combining and partitioning

Using fingers to represent and show connections between numbers

Purpose

To make and reinforce connections between a "number less than 20" and 5 and 10.

Rationale

In order to enable students to use efficient strategies when adding and subtracting, they need to be familiar with relating small numbers to 5 and 10. For example, the strategy of "bridging" (e.g. 8 + 5: 8 + 2 = 10, 10 + 3 = 13) requires students to see that:

(a) 8 requires 2 more to make 10,

(b) 5 - 2 = 3, and

(c) 10 + 3 = 13.

This series of activities is designed to assist the initial step of building to ten.

A major task is to help students become automatic with pairs of numbers that make 10, sometimes referred to as "complementary numbers". The activities also incorporate a second and less often stressed series of connections, those with the number 5. For example, in realising that 7 requires 3 more to make 10, some students are helped by knowing that:

(a) 5 + 5 = 10, and

(b) 7 = 5 + 2.

One reason for emphasising this connection is that, for some students, 6, 7, 8, 9 are already quite large numbers. The other, and parallel reason, is that students have two hands, each with 5 fingers, and this connection, if made explicit, provides a very powerful model.

The following activity is an important preliminary to using the Chinese abacus, which requires knowledge of connections with 5.

Combining refers to bringing parts together; *partitioning* refers to separating the parts while maintaining the sense of the original number. It emphasises part–whole number relations, e.g.

3 + 2 = 5

5 - 2 = 3

5 - 3 = 2.

Stage 3

Activity 1

***Counting On* framework reference: Level 0**

Use the following routine, varying the number between 0 to 5 (3 is used as an example). At first go very slowly, gradually increasing speed as long as students can keep up.

Step	Teacher says:	Students say:	Actions and comments
1	"Show 3 fingers."		Students raise 3 consecutive fingers of one hand. You may insist that fingers are always shown starting from the little finger of the left hand.
2	"How many more to make 5?"	"2"	Students may check initially before replying by seeing how many of the fingers of their left hand are bent down.
3	"What makes 5?"	"3 and 2 make 5"	
4	"5 take 3 is...?"	"2"	
5	"5 take 2 is...?"	"3"	

Activity 2

***Counting On* framework reference: Level 0**

Use the following routine, varying the number between 5 and 10 (6 is used as an example). At first go very slowly, gradually increasing speed as long as students can keep up.

Step	Teacher says:	Students say:	Actions and comments
1	"Show 6 fingers."		Students raise 5 fingers of one hand and 1 of the other. Encourage students to use all the fingers on one hand for this activity.
2	"6 is 5 and...?"	"1"	
3	"How many more to make 10?"	"4"	Students may check initially before replying by seeing how many of the fingers of their hand are bent down.
4	"What makes 10?"	"6 and 4 make 10"	
5	"10 take 6 is...?"	"4"	
6	"10 take 4 is...?"	"6"	

(Before moving to Activity 3, which uses the previous activity as a basis for adding two single digit numbers by "bridging", it may be helpful to use the *Grids to ten* activity. These will provide another set of visual clues for students.)

Activity 3

***Counting On* framework reference: Level 0**

Check that students know instinctively the result of adding a single digit to 10: for example, that 10 + 6 = 16. If there is any doubt, this problem needs to be dealt with before proceeding with Activity 3.

Use the following routine, varying the first number between 6 and 9 (8 is used as an example) and the second number between 1 and 9 (7 is used as an example), providing the sum is a two-digit number. You might like to follow this activity by recording partitioning on the board, e.g.

$$8 + 7 = 8 + \overset{\wedge}{7} = 8 + 2 + 5 = 10 + 5 = 15$$
$$(5+2)$$

At first go very slowly, gradually increasing speed as long as students can keep up.

Step	Teacher says:	Students say:	Actions and comments
1	"Show 8 fingers."		Students raise all fingers of left hand and first 3 fingers of right hand.
2	"8 is 5 and...?"	"3"	Students should no longer need to check by seeing how many of the fingers of their right hand are bent down.
3	"How many more to make 10?"	"2"	
4	"How many more do we need to add?'	"5"	
5	"How did you work that out...?"	"8 and 2 is 10 and 5 more makes 15"	Any response that shows that the child used the bridging strategy is acceptable. There are other equally good strategies, for example some students, if asked to add 8 + 7, may use a near doubles strategy and say: "I know that 7 + 7 is 14, so 8 + 7 must be 15". It is important to establish in this context that you are practising the "bridging ten" strategy and so this strategy is to be used for the moment. There should be no suggestion that the near doubles strategy is not equally good at other times.

Later you may omit Steps 1 and 2 and start with Step 3. You may also ask sometimes, after Step 4: "Why did you add 5 more?" in order to encourage the child to articulate and so consolidate the reasoning, for example: "5 and 2 make 7, and I had already added 2 to make 10".

Grids to ten

Stage 3

Counting On **framework reference: Level 0**

Purpose

To consolidate the recognition of pairs of digits which total 10.

Rationale

Consolidation activities are needed to cement and maintain skills which have been acquired. This activity can follow activities such as the finger, or ten-grid activities or others whose purpose is to build up knowledge and the ability to recall pairs of numbers whose sum is 10.

Materials needed

One 5 x 5 board and 24 counters per student (or a pencil, if numbers are to be crossed out). Teachers may use the blank grids to make up further puzzle grids, or students may be challenged to make up one for a friend to solve. Grids will need to be enlarged for counters to fit.

Time required

5–15 minutes.

Activity 1

Explain that students are to cover with counters, pairs of numbers whose sum is ten. Alternatively, pairs of numbers can be crossed out with a pencil. One number will be left uncovered, and this number is to be recorded beside the grid. Students work individually at one or more of the grids.

1	6	7	2	2
9	8	3	4	4
5	7	4	9	2
6	6	7	4	5
3	8	8	3	1

7	1	2	5	4
4	2	6	8	1
7	7	6	3	2
6	8	9	5	3
5	9	3	5	8

1	2	3	4	5
0	1	6	7	6
3	2	5	6	7
4	3	4	5	8
5	6	7	8	9

Addition pairs to 10

Counting On **framework reference: Level 0**

Purpose

To develop confidence in the addition of pairs of numbers to 10.

Materials needed

At least four sets of numeral cards for each pair of students. For example, for the Make 7 Fish activity, numeral cards 0–7 are used whereas for the Make 9 Fish activity, numeral cards 0– 9 are used.

Counters and lids, recording sheet.

Instructional sequence

1. Whole class

   Using two circles on the overhead projector, ask: *Who will volunteer to put these seven counters in the two circles*?

   After a student places the counters, ask: *Who could arrange the seven counters differently*?

2. Pairs or individual

   Students are given two lids (or pieces of cardboard) and asked to find as many different ways as possible to arrange the seven counters on the two lids and record the answers.

3. Whole class
   (a) Students report on their arrangements.
   (b) How many different pairs are there?
   (c) Extension: On the overhead projector, hide 4 counters under one lid and 3 counters under the other lid. *If there are four counters under this lid, how many will there be under the other lid*?

4. Small group activities
   (a) Fish
   (b) Ten pin bowling.

Activity: Fish

***Counting On* framework reference: Level 0**

Purpose

To consolidate the recognition of pairs of digits which total 7.

The game is played like traditional Fish with players making pairs of cards adding to the given number. For example, with Make 7 Fish they make pairs adding to 7.

Materials needed

Four sets of numeral cards 0–7 for each pair of students

For those unfamiliar with Fish, here is a description of Make 7 Fish:

- Five cards are dealt to each player.
- The remainder of the cards are put in a stack in the middle of the table.
- Students make pairs of cards adding to 7.
- Pairs are placed in front of each player.
- Students take turns to ask another player if they have a certain number. For example, if a 2 is needed to pair with a 5, the player asks another student, "Do you have a two?"
- If the player has a 2, then this is given to the player making the request and the resulting pair is placed on the table.
- If the player doesn't have a 2 he or she says "Fish" and the player who made the request takes the top card from the stack.
- The game concludes when a player has no cards.

Activity: Ten pin bowling

***Counting On* framework reference: Level 0**

Purpose

To develop confidence in counting tens

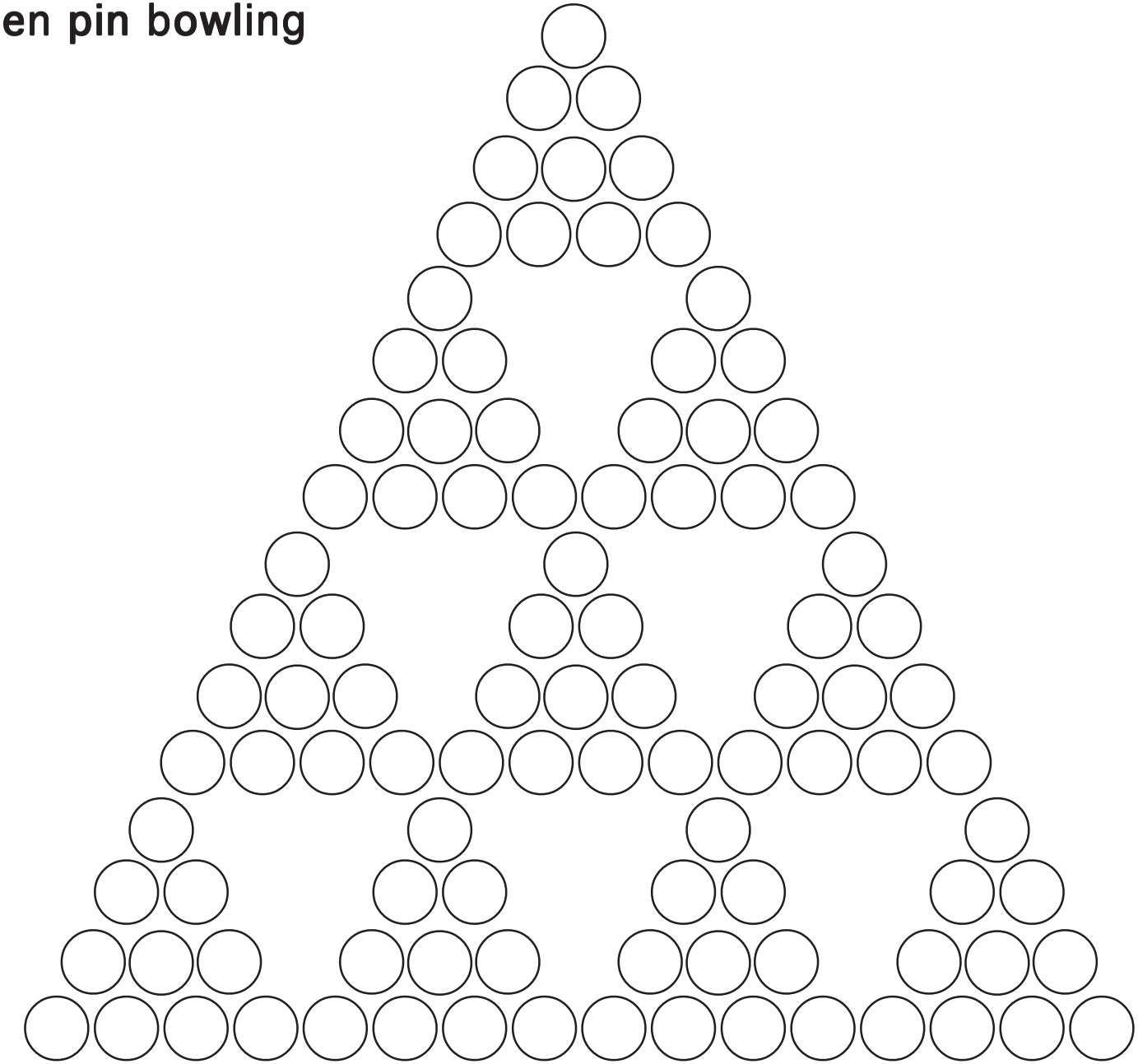
Materials

Sets of digit cards 0–9, twenty cards with the word *STRIKE* printed, ten pin bowling baseboard, counters or pencils to colour the circles. If counters are to be used, the proforma will need to be enlarged to A3.

Instructions

- Each player has a baseboard and pile of counters.
- The digit cards are mixed with the STRIKE cards and placed in the centre.
- For each frame a player takes 2 cards and adds the numbers. The total number of counters is placed on the board. If the player turns over a STRIKE card, he or she selects another card and doubles their total score for the round.
- If at any time a player can make a ten with 2 turns this is called a SPARE and they have another turn.
- After 5 turns each, the winner is the player with the highest total.

Ten pin bowling



Write a story

Counting On **framework reference: Level 0**

Purpose

To help students link number facts to appropriate questions.

Materials needed

Questions on cardboard, or questions written on the board.

Activity 1

1. Provide students with a number sentence and ask them to generate a question that relates to it. Begin with questions relating to addition and subtraction, and then extend to questions which relate to multiplication and division. The emphasis should be on the question rather than on the answer.

2. Show the following addition and subtraction cards or write the expressions on the board and ask students to think of a question that matches the expression.

7 + 5	8 – 3

   Encourage a variety of responses, such as "I have seven cakes and I buy another five", "My sister had seven apples, then I gave her five more", "I had 8 computers and three broke down", "There were eight birds and three flew away".

3. Show expressions with multiplication and division statements and ask students for more stories.

5 x 4	12 ÷ 3

   Encourage responses such as "There are five chairs and each has four legs", "Five cats have four kittens each. How many kittens?", "I can see twelve legs and each stool has three legs. How many stools?"

Extension

Change cards to include a sequence of operations, e.g. 7 + 5 - 3 + 1.

Notes

Unit 2: The Chinese abacus

Purpose

To reinforce place value concepts, including zero as a place holder; to use composition of numbers in relation to 5 and 10; to move away from counting on and back in ones; to engage in multi-digit addition (and later subtraction) and mental computation.

Rationale

There are several reasons for using the Chinese abacus as a tool for developing arithmetical skills and understandings in the classroom. First, it helps to develop some of the most fundamental ideas and skills, including place value, addition and subtraction, and mental computation.

Second, it approaches these ideas from a fresh angle.

A third reason for using the Chinese abacus in the classroom is that it is not designed only for learning purposes in schools. The abacus is an adult tool, the equivalent of an electronic calculator, which is used today in various forms in many Asian countries.

The purpose of these activities is not to make students either expert or exceptionally fast in the use of the abacus. Some of the technical details, for example the correct fingers to use, are not considered necessary at this stage. Most students should learn to enter and read numbers, understand how to perform 3-digit or 4-digit additions and subtractions, and achieve some success in doing so.

Background for teachers

Introduction

This section provides background information on using the Chinese abacus. This section is lengthy while the subsequent descriptions of student activities are relatively brief.

Description of the Chinese abacus

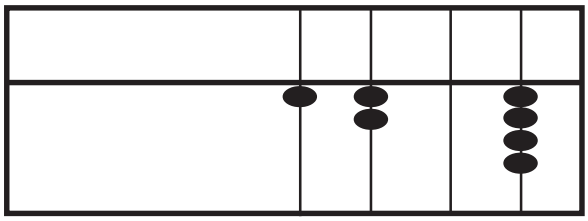
The Chinese abacus is used in landscape orientation, i.e. lengthwise, and laid on a horizontal surface. Each bamboo column has two beads above the central wooden “beam” and five beads below the central beam. We will refer to these as upper and lower beads respectively. Each of the five lower beads in any column is worth one (units or tens or hundreds ...), while each of the two upper beads is worth five (units or tens or hundreds ...). However, no bead has any value unless it is pushed towards the central beam.

Unless at any point you wish to introduce decimals, take the right-hand bamboo column to represent units, the next column tens, the next hundreds, and so on.

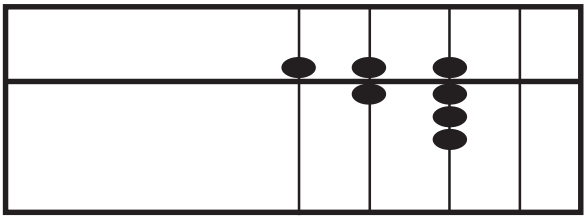
Entering and reading numbers

To start, all beads should be pushed up to the top rim or down to the bottom rim, away from the central beam. This is called clearing the abacus.

In the following figures, only beads touching the central beam are shown.



This abacus shows 1204.



This abacus shows 5680.

Now spend some time entering and reading numbers, preferably working and checking with someone else, until you can enter and read up to 4-digit numbers easily.

You will notice how the abacus shows the need for a zero to act as a place holder when recording numbers in symbols. The upper beads are valuable in allowing you to read a column at sight. If there were only 10 lower beads and no upper beads, it would be very difficult to distinguish between 7, 8 or 9 beads. This also links very clearly to the way one represents numbers by using fingers: 8 is 5 (one hand) plus 3 (fingers on the other hand).

Exchanging

Before starting with addition, you need to note some situations involving exchanges. These are the same for every column so we will consider only the units column.

1. Push one lower bead at a time up to the central beam and count: 1, 2, 3, 4, 5. At this point we have all five lower beads in use, and the count is 5. But each of the upper beads above the beam is worth 5, so we can exchange. Push the five lower beads down away from the central beam and push one upper bead down to the central beam. The number shown is still five, but the exchange has made more economical use of the beads.

2. Continue to push the lower beads in the first column up to the beam while counting: 6, 7, 8, 9, 10. Again we have all five lower beads up against the central beam, so again we can exchange. Move the five lower beads down, and move the second upper bead down. The number shown is still 10, made from the two upper beads.

3. However we have still not finished the exchanges. Each of the lower beads in the second column is worth 10, so we move the two upper beads in the units column back up, and, in exchange, move one lower bead in the tens column up to the beam.

4. This was in fact an unnecessarily slow way to achieve this. We could have seen that, when we had one upper bead and the five lower beads against the bar in the units column, the abacus was showing 10. We could then have moved all those beads away from the central beam and moved the one lower bead in the tens column up. However if we are not worried about speed, it may help to make these incremental changes until we feel secure about more sophisticated ones.

Adding on the abacus

Adding two numbers of any size is performed by entering the first number on the abacus, and then adding on each digit of the second column, starting with the units.

Most of the difficulties are resolved when learning to deal with the first two columns and we will concentrate on these.

An example: Adding 3 + 4

It is very important that we use the abacus to perform the calculation. For example, when adding 3 + 4, we should not, after entering the 3, calculate mentally that 3 + 4 is 7, and therefore focus on entering 7. It is important that, after entering the 3, we think: "I want to enter 4 but I have only two available lower beads. I can use one of two strategies; either I can add those 2, perform an exchange of one upper for the five lower beads, and now enter the remaining 2 (lower beads). Alternatively, I can replace '+ 4' with '+ 5 - 1' ('adding 4' by 'adding 5 and subtracting 1'). I add one upper bead and remove one lower bead".

It is worth going over this until the problem and the two alternative solution strategies are quite clear. It is this kind of thinking that lies at the heart of the value of using the Chinese abacus with students. Over and over again one has to think of simple equivalencies (in this case + 4 as either + 2, + 2, or as + 5, - 1), in order to calculate. In neither case do we mentally calculate the answer 7: we perform the addition and then read the answer 7 from the resulting position of the beads on the abacus.

Adding two single-digit numbers

We will first try adding some pairs of single-digit numbers to consider the range of problems and solution strategies that arise. Two examples of each type are given.

1. Add 1 + 3; 2 + 2. There are enough lower beads.
2. Add 5 + 2; 3 + 6. There are enough available upper and lower beads.
3. Add 3 + 2; 4 + 4. In each case, two possible strategies, as explained above are used: either add lower beads until you have used all 5, then exchange, then add the remaining lower beads or, add one upper bead and subtract the requisite number of lower beads.
4. Add 7 + 5; 8 + 6. Add, and then exchange.
5. 4 + 9, 8 + 8. While other ways are possible, the simplest is to add one ten (i.e. one lower bead in the second column, and then subtract 1 or 2.)
6. 8 + 7; 9 + 6. It is easier to add the relevant beads in the units column, and then deal with the necessary exchanges.

These represent all the types of addition situations. It is important to remember that while there may be a "neater" way of dealing with these sums, there are often several strategies to use. Any strategy which works should be encouraged. It is definitely not a question of teaching the "best rule" for each situation.

Set yourself a variety of single-digit additions and consider and discuss different solution strategies.

When you feel confident, try the following: Add successively 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10. In doing it you will face all the addition situations.

Adding multi-digit numbers

The general procedure is to enter the first number and then to add the second number, one digit at a time, starting with the units column. One can start with the left-hand digit instead, and sometimes that proves easier. However it is suggested that students start by adding from the right, record their answer in symbols, and then do the same calculation starting from the left as a check on their answer.

Practise adding some 2-, 3- or 4-digit numbers with a colleague, and compare strategies and any difficulties or uncertainties that arise.

One traditional series of exercises starts by adding 12 + 12 + 12... (ten in all). It is easy to see by inspection if your final answer is correct, since the abacus should show 120.

The series of exercises continues by adding ten lots of 123, then ten lots of 1234, and so on as far as you wish to proceed up to ten lots of 123456789.

Entering and reading numbers

Activity 1

Counting On **framework reference: Level 0**

Materials needed

One Chinese abacus per student.

Time required

Two or three lessons.

Instructional sequence

1. Introduce the abacus to the students. Address the following areas: How to place it; value of columns; value of beads; use of the central beam; starting position for beads
2. Show how to represent and read single-digit numbers. Highlight the connection with fingers.
3. Explain how to enter and read two-digit numbers, and how to enter multiples of ten, i.e. zero as place holder.
4. Show how to enter and read larger numbers. Four-digit numbers representing years, or larger numbers, such as phone numbers could be entered for other students to read.

Adding single-digit numbers

Activity 2

Counting On **framework reference: Level 0**

Materials needed

One Chinese abacus per student.

Time required

One lesson

Instructional sequence

1. Revise how to enter and read a single-digit number. Students enter the digits 0 to 9 on successive columns starting from the left column, focusing on this as a sequence of digits, not as a multi-digit number. Discuss the patterns.
2. Follow the sequence in Activity 1 above. Where possible give each addition as a challenge for students to try, and ask them to describe their solution strategies.
3. Give a list of pairs of digits for students to add, perhaps recording their answer as a drawing of beads touching the centre, and recording their strategy.

Adding larger numbers

Activity 3

Counting On **framework reference: Level 0**

Materials needed

One Chinese abacus per student.

Time required

Two or more lessons.

Instructional sequence

This must be determined by the ability level and interest level of the students. Where possible, work sheets should be given as homework for students to have time to work on at their own pace. Class time should be used for introducing new steps, discussing possible solution strategies, consolidating and practising, and, only where appropriate, providing challenges involving larger numbers, or speed competitions.

Subtraction

Activity 4

Counting On **framework reference: Level 0**

Materials needed

One Chinese abacus per student.

Time required

To be determined.

Instructional sequence

A similar sequence of lessons can be developed for subtraction on the Chinese abacus, and the difficulty level is only slightly higher than for addition. Since the principles are the same as for addition, some students might be challenged to try to develop procedures for subtraction. The focus should be on the exchanges of units required.

Unit 3: Ten-frames

Stage 3

Related concepts

5 as a base, combining and partitioning, bridging 10.

Rationale

The ten-frame assists students to form a clear understanding of the base ten structure of numbers. When ten-frames are used in conjunction with counters, students can model combining and partitioning in a structured way. For example, 3 counters and 4 counters can be combined to show a total of 7 counters.

Just as ten is essential to our understanding of operations on numbers, five often acts as a base for mental calculation. This is due to students' early use of finger strategies with arithmetic. The ten-frame is organised as five squares and five squares, which replicates the first organised material students use, namely, two hands with five fingers on each.

Materials needed

Sets of ten-frames. This includes one set for the overhead projector and multiple sets for small group or partner activities.

Numeral cards.

Instructional sequence for ten-frames

Whole class activities:

- Flash the cards on the OHP. Students can name the number and state how they worked it out. For example, 10 means all spaces filled or 7 is 5 on the top plus 2 on the bottom.
- Place 10 on the OHP and cover. Add other numbers for students to add on such as 6, 7 or 9.
- Place 7 on the OHP and cover. Keep adding tens for students to practise counting off the decade.
- Place 2 empty ten-frames on the OHP. Put 6 counters in one frame and 5 counters in the other. Ask students to suggest a good way to find the total.
- Place dot ten-frames on the OHP and ask students to explain how they found the total.
- Expand the concept by using ten-frames with 20, 30, 40 or 50 on cards.

Partner activities:

- Place the pack of ten-frames between a pair of students. Students take turns to turn over the top card and state the number. Students should be able to state how they instantly recognised the number.
- Have students take turns to turn two cards over and state the total. The partner is responsible for checking the answer. The totals could be recorded and tallied.

- Students work with a partner to make a nominated number such as 15. The challenge could be to make the number with two cards only, e.g. 10 + 5, 9 + 6 or with many cards. Sequences could be recorded on the board for discussion and investigation, e.g. 6 + 3 + 2 + 1 + 1 + 2 = 15
- Students, in pairs, challenge each other to make a number
- Students take turns to place two cards face-down side by side. The student, who has looked at the cards, turns one card face up and says "The total is…….., what number is hidden?"
- Three cards are dealt to each player. The aim is to make 10 using two of the cards. Addition or subtraction can be used. One card can be discarded each round and another card picked up from the pack. The winner is the player with the most pairs.

Number combinations to twenty

Purpose

To help students recognise "five as a unit".

***Counting On* framework reference: Level O.**

Materials needed

Cards with various sums, where the addends are five or greater. Refer to proformas on pages 38–40.

Grids with various numbers represented by dots. Refer to proforma on page 37.

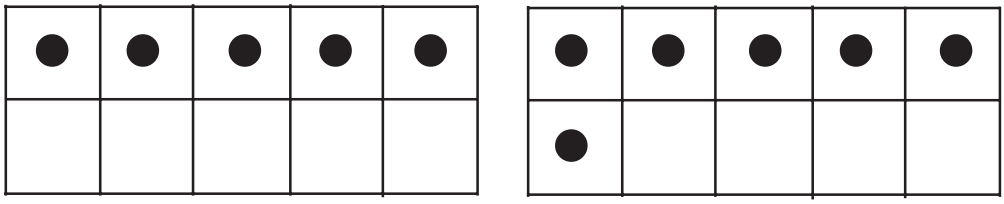
Time required

One lesson.

Instructional sequence

Activity 1

1. Turn over a card which shows a sum, say 5 + 6. The total of the two numbers should be 20 or less. Find a corresponding five-plus card for each number. In this instance, select the following two ten-frames.

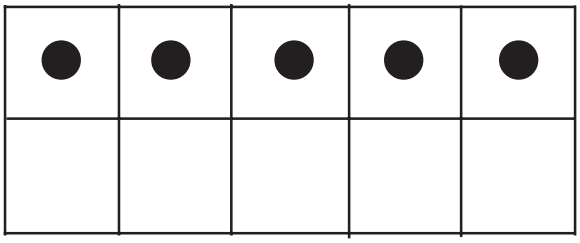
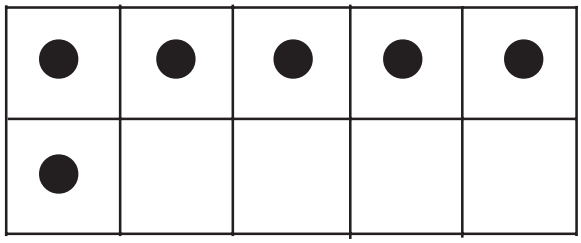
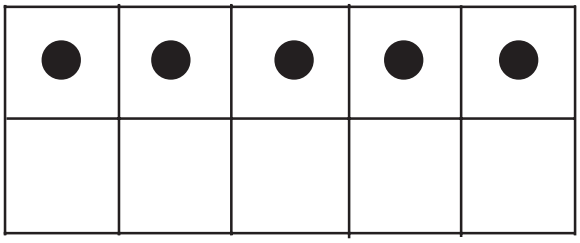
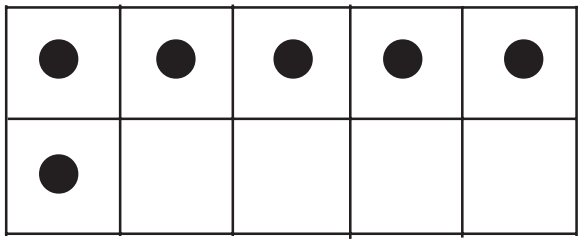
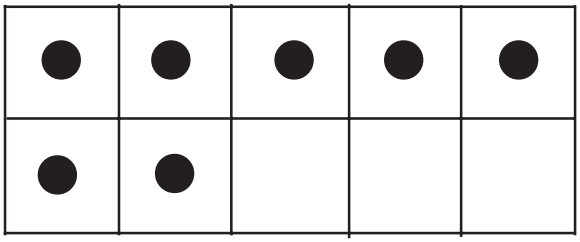
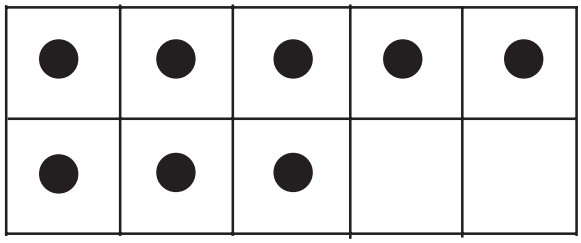
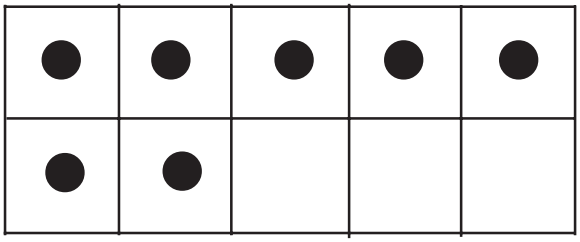
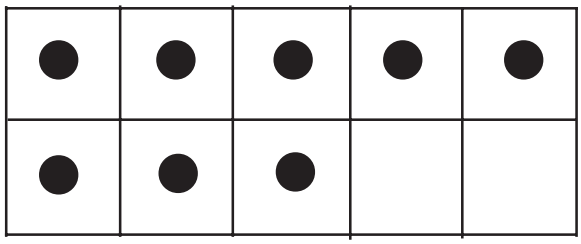
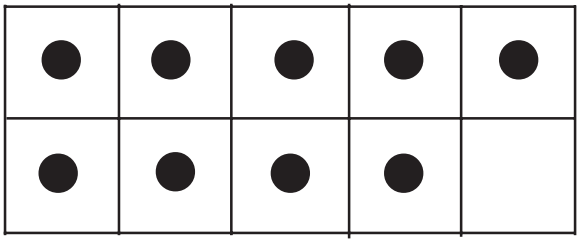
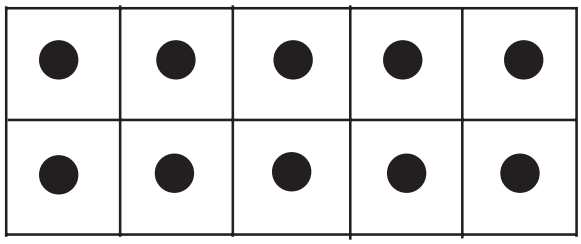
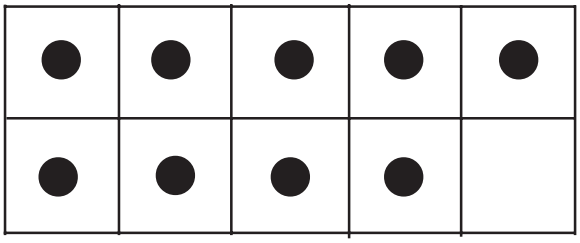
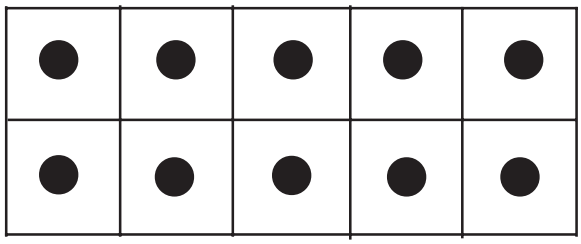


2. Encourage students to mark each strip of 5 dots with a highlighter. This helps students recognise the "five" as a unit. Students should add these as 5, 10, and 1 is 11.

   Discourage a count by ones strategy.

3. Repeat the process. Each time the students must select the ten-frames to represent the numbers and count the "fives" first.

Five-plus cards

5 + 6	6 + 5
7 + 6	5 + 7
7 + 8	7 + 9
6 + 9	8 + 8
5 + 8	7 + 10
7 + 7	9 + 9

7 + 5	8 + 5
6 + 7	8 + 7
9 + 6	5 + 9
5 + 10	9 + 7
9 + 5	6 + 6
5 + 5	8 + 6

8 + 9	9 + 10
6 + 8	8 + 10
6 + 10	9 + 8

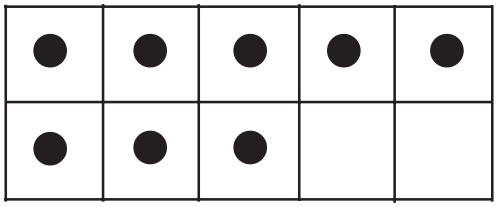
Dice

Activity 2

***Counting On* framework reference: Level 0**

1. Using two blank dice, attach or write the numbers on each side from 5 to 10. If blank dice are not available, attach "sticky labels" to standard dice.

2. Roll the two dice, then select the five-plus card that represents the larger number and then add. For example, if the numbers 7 and 8 are thrown, select the following ten-frame and encourage the student to find the total in the following way.



"8 and 2 make 10 and 5 more is 15, so the total is 15". Discourage counting by ones.

Five plus combinations

Activity 3

***Counting On* framework reference: Level 0.**

Purpose

To develop knowledge of number combinations involving five.

Materials needed

Five-plus cards for the overhead projector

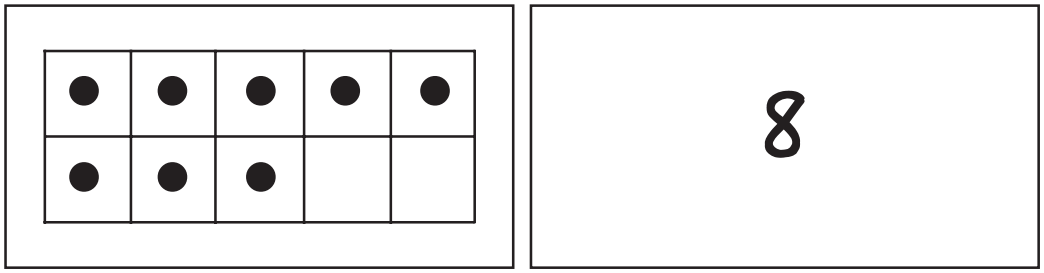
Five-plus cards

Numeral cards to match *Five-plus* cards.

Instructional sequence

1. Whole class
   - Flash a *Five-plus* card on the overhead projector. How many dots?
     Repeat for other *Five-plus* combinations.
   - Tell me what the *Five-plus* card for 8 would look like.

2. Small group (or pairs)
   Five-plus fish
   This is played like the traditional game of Fish using *Five-plus* cards and numeral cards. Students make pairs of the *Five-plus* and its matching numeral card.



Variation

A game of Snap could be played using the five-plus and numeral cards.

Adding whole tens

Activity 4

***Counting On* framework reference: Level 1**

Purpose

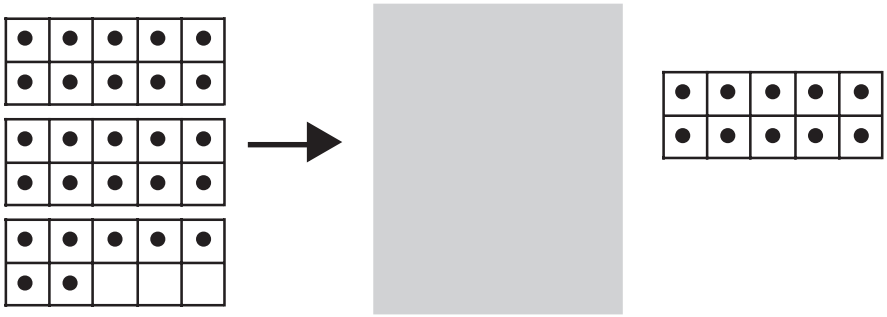
To develop confidence in addition of tens using a collection–based strategy. This strategy is described in detail on p. 67.

Materials needed

Ten-frames for overhead projector (use proforma on p. 55), adding ten cards.

Instructional sequence

1. Whole class
   - Display 27 using ten-frames. There are 27 dots. Next to the screen place another ten.

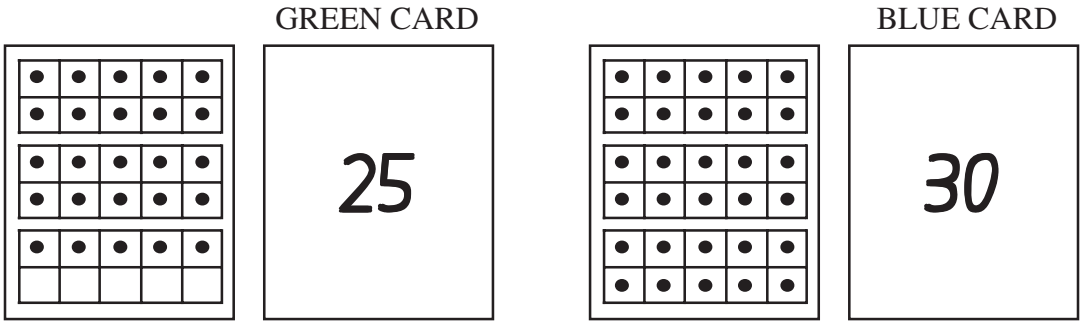


How many tens are there? How many dots altogether?

Repeat this for other additions of whole tens.

2. Small group

   Adding tens game.
   - Pairs of students
   - A set of green cards with the numbers 12 to 59 printed on them and a set of blue cards showing 10, 20, 30 or 40. A set of ten-frames from the proform on p. 55.



   - Students in pairs take turns to select a green card, find the matching ten-frame, and record the number on the log sheet. They then select a blue card, find the matching ten-frame, and record this number on the log sheet.
   - The student then records the number of tens and the number of dots on the log sheet.

Adding tens log sheet

Name: _____

Green card	Blue card	No. of tens	Total number of dots
Example 25	30	5	55

Race to 30

Activity 5

***Counting On* framework reference: Level 1**

Purpose

To help students to "see" the strategy of building to the next multiple of 10 to solve addition and subtraction problems.

Materials needed

Set of A4, ten-frame flash cards; set of A4, double ten-frame flash cards (these can be made using the proforma on page 51); dice (1–6): one for each pair of students; counters: 30 per student; game mat: 3 empty ten-frames: one per student (refer to the proforma on p. 47).

Time required

Two lessons.

Instructional sequence

1. The teacher flashes a sequence of ten-frames. Students announce how many dots and how many more to make 10.

2. The teacher then flashes double ten-frames. (The top ten-frame is always full. This is a strong representation of the ten in the numbers 11 to 20). Students announce how many dots altogether and how many more to make 20.

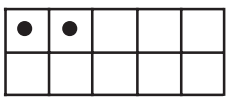
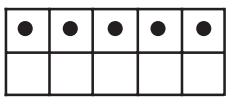
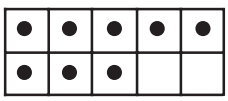
3. The teacher flashes a double ten-frame such as 17. The teacher then says "*and 5 more*". With this strong visual representation many students are likely to use this strategy to solve this problem: 17 + 3 = 20; 20 + 2 = 22.

4. Play the game *Race to 30*.

Rules

1. Students play in pairs. They take turns to roll a 1–6 die. When a number is rolled, a student places that number of counters on his or her work sheet.

2. Students must fill each ten-frame from the top left. The top row must be filled first,

e.g. 2 =  5 =  8 = 

After each turn, the student must state how many counters are on his or her chart and how many more are needed to make 30.

3. Students fill the top ten-frame first, then the middle ten-frame and finally the bottom ten-frame.

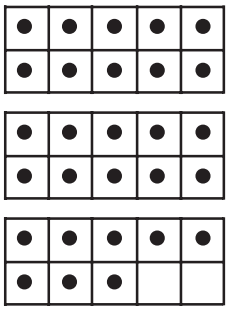
   The first student to reach 30 wins. It is not necessary to get the correct number to finish.
4. At any time the teacher may ask a student *What's your score and how many more do you need to make 30*? If the student cannot answer, he or she misses a turn. A student whose score is say 16, should be able to see that 10 + 6 = 16, 4 more are needed to make 20 and 10 more will make 30. Therefore, the response should be *My score is 16 and I need 14 more to make 30*.

Variation

This game may be used to teach subtraction across multiples of 10. For example, 24 – 6 can be solved with this strategy as 24 – 4 = 20 and 20 – 2 = 18.

To play this variation of the game students fill all three ten-frames with counters. They then take turns to roll dice. They take the number they roll from the board starting in the bottom row of the bottom ten-frame, e.g. 30 – 2 =

In this variation the correct number is not needed to finish.



Race to 30

Adding tens and ones (1)

Activity 6

***Counting On* framework reference: Level 1**

Purpose

To develop knowledge of collection–based addition strategies for tasks that don't involve re-grouping.

Materials needed

Ten-frames, use the proforma on p. 55. Make a set for the overhead projector and extra sets for students.

Log sheet, use the proforma on p. 49.

Instructional sequence

1. Whole class
   (a) Use the ten-frames to make 41. Use the ten-frames to make 35 and put next to 41.
   How many tens are there?
   How many extra dots are there?
   How many dots altogether?
   Give similar tasks.
   (b) Using ten-frames, display and then screen 34.
   There are thirty-four dots.
   Place another 23 next to the screen.
   I put out another 23.
   How many tens are there?
   How many extra dots are there?
   How many dots altogether?
   Give similar tasks.

2. How many?
   Students in pairs or individually play the game of determining *How many?*
   Green cards: 31–35, 41–45, 51–55, 61–65
   Blue cards: 21–24
   Cards can be made from the proforma on p. 109.
   Students complete the Log sheet. Students may use ten-frames as visual prompts.

3. Whole class
   Teacher displays one green card and one blue card, e.g. 65 and 24. Students solve as a mental computation. Students share strategies.

How many? Log sheet

Name: _____

Green card	Blue card	Tens	Ones	Total
Example 41	35	7	6	76

Adding tens and ones (2)

Activity 7

***Counting On* framework reference: Level 1**

Related concepts

Hundreds chart, empty number line.

Purpose

To develop knowledge of counting–based addition strategies for tasks that don't involve re-grouping.

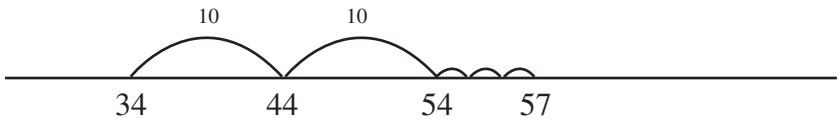
Materials needed

Hundreds chart transparency

Ten-frames.

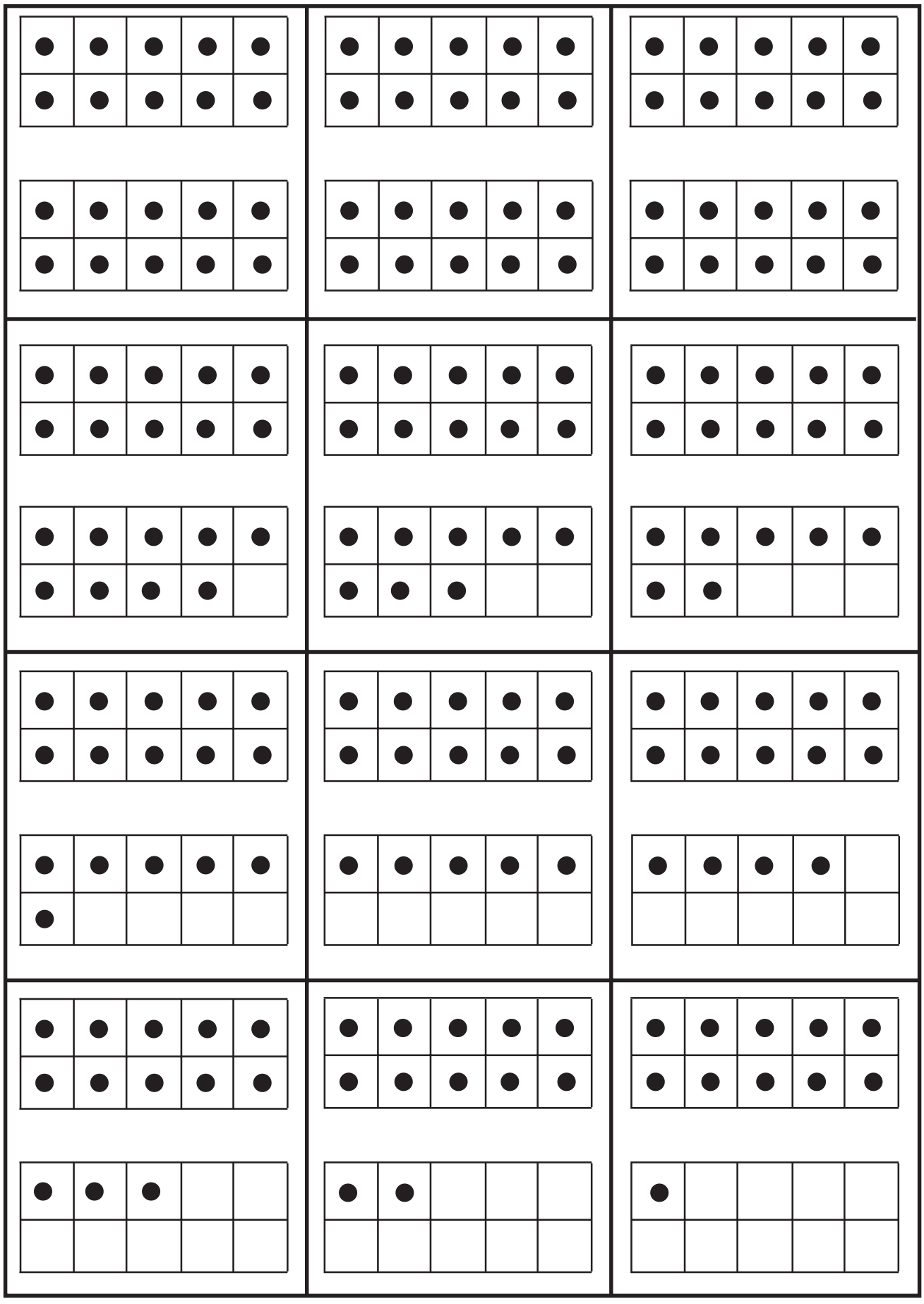
Instructional sequence

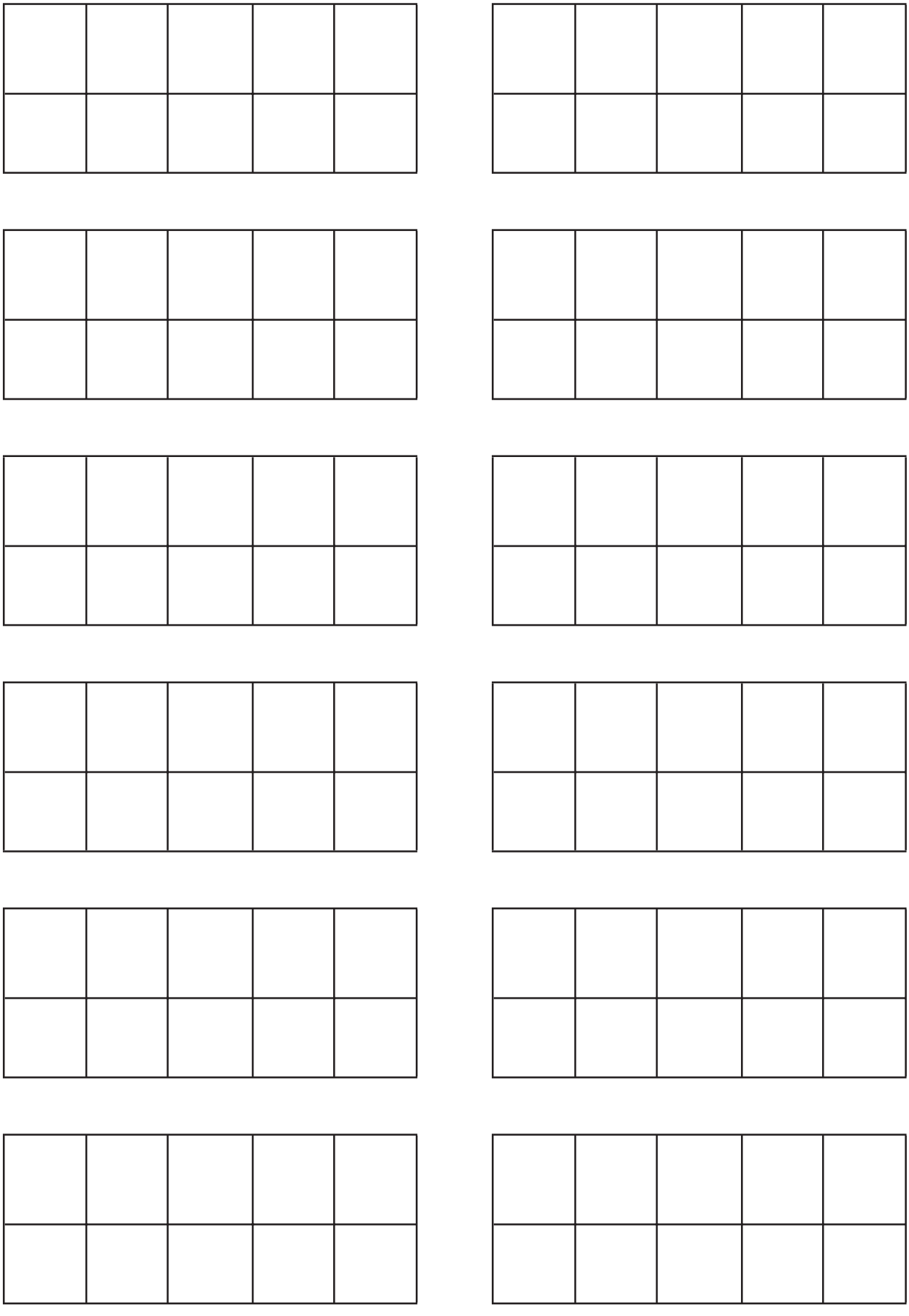
1. Whole Class
   (a) Display a hundred chart on the overhead projector.
   Start at four and count by tens down the four column. Four, fourteen, twenty-four, ...
   Repeat starting from other numbers.
   (b) Count by tens from different starting points without viewing a hundreds chart.
   Start counting by tens from thirty-two.
   (c) Using ten-frames display and then screen 34.
   There are 34 dots. Display another 23 dots. *Here are another 23 dots. How many altogether*?
   Ask students to discuss their solution strategies.
   Map each strategy on an empty number line.
   If a student adds two tens and then another 3, it would be mapped as follows.

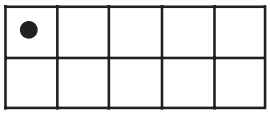
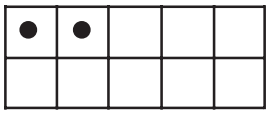
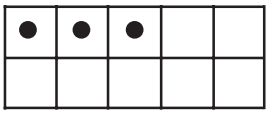
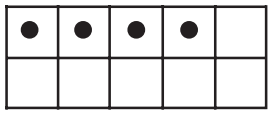


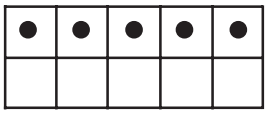
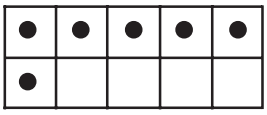
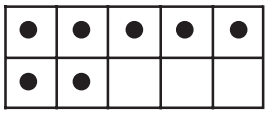
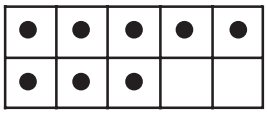
   Give other addition tasks and map the students' strategies.
   (d) Give further addition tasks and have students map their strategies. Have students share their maps with the class.
   (e) Show the class a mapping and ask the class to comment on the task that has been asked and the solution strategy.

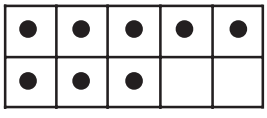
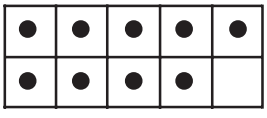
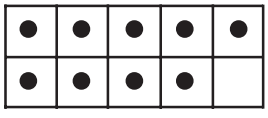
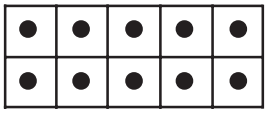
Note: The sequence above can also be used for subtraction.

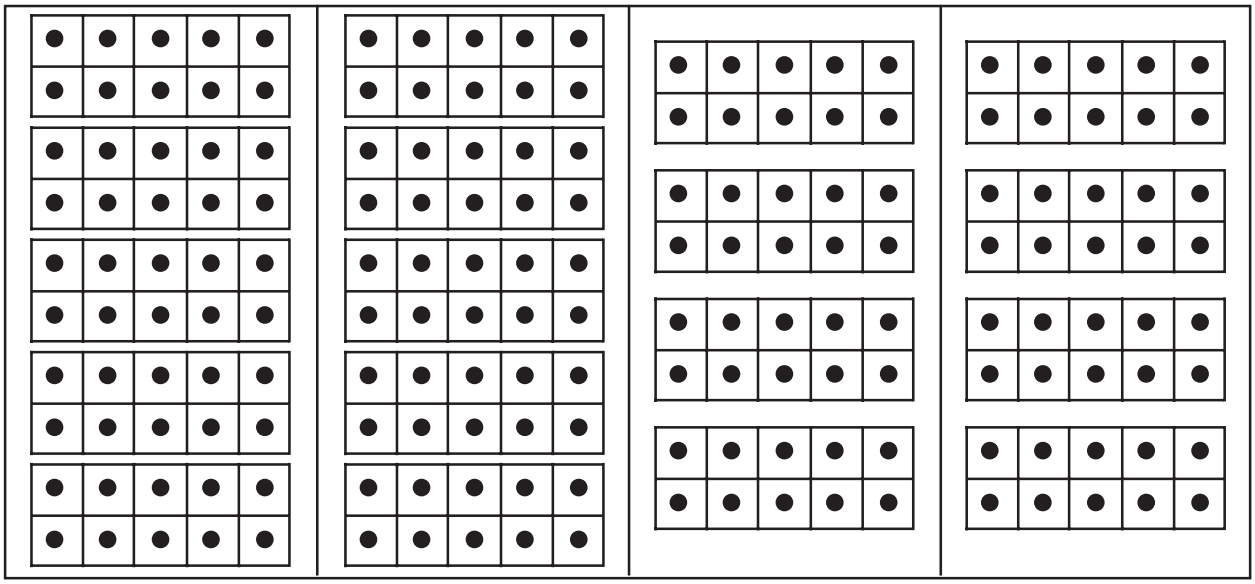


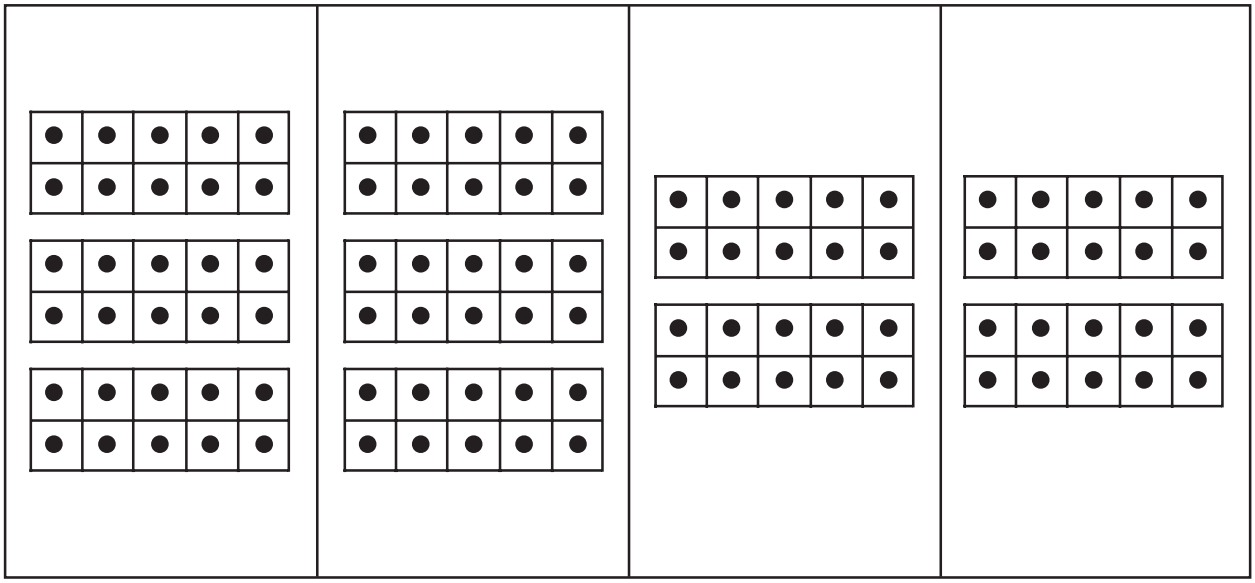


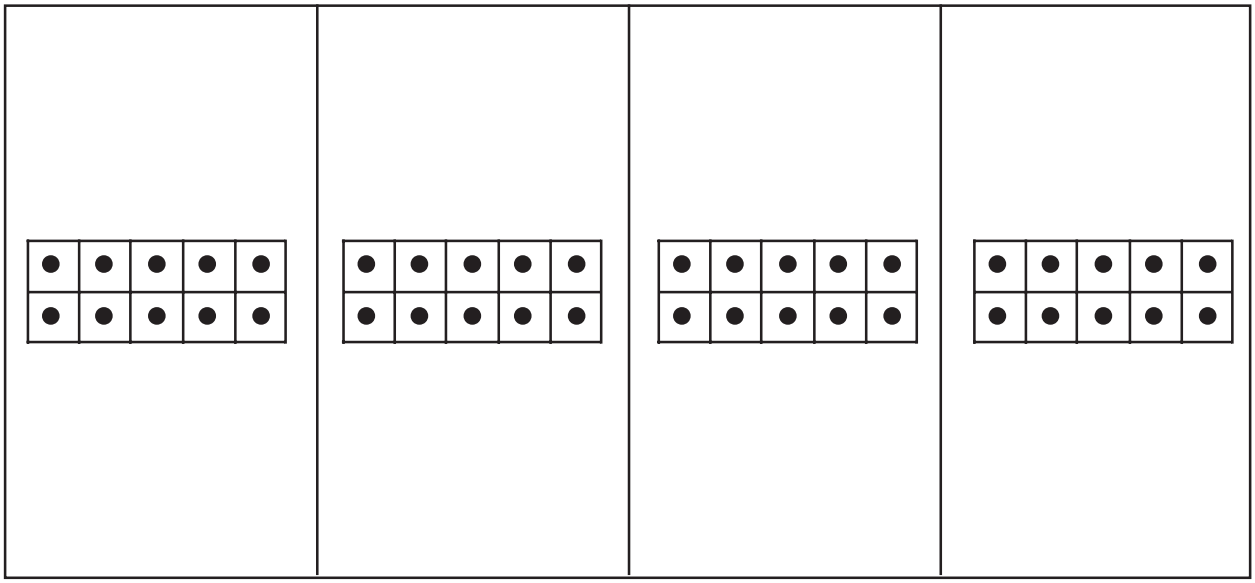





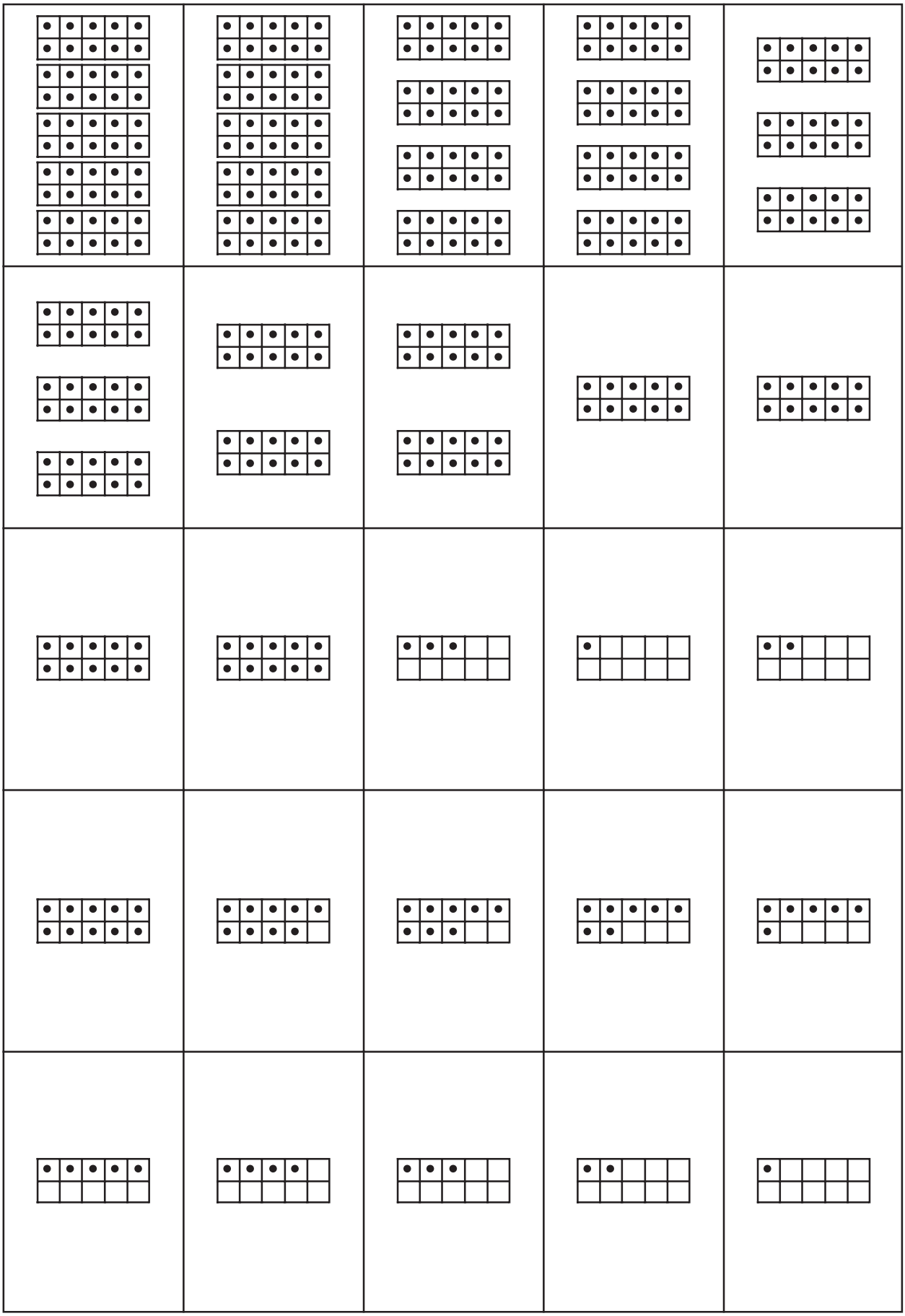





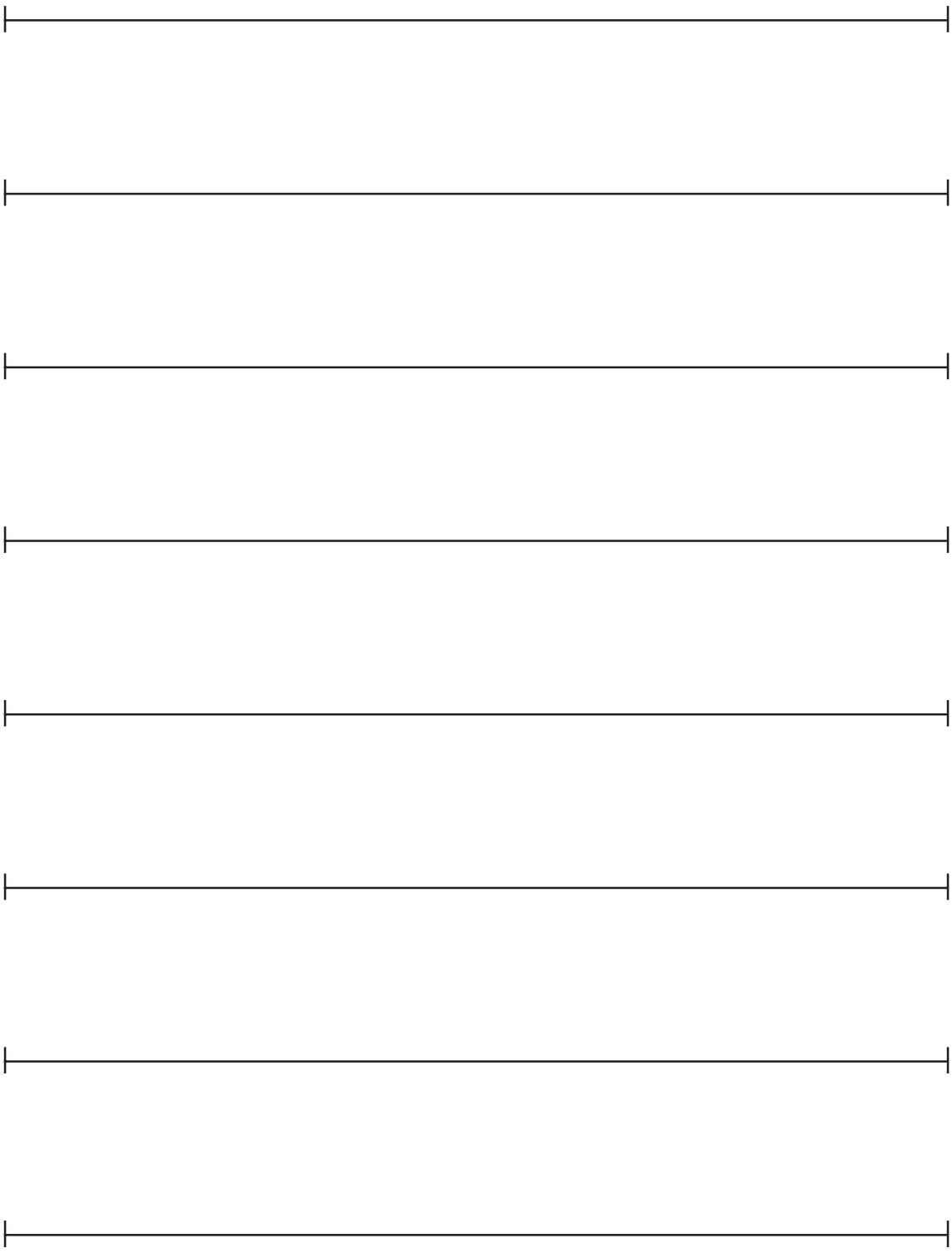








Empty number line proforma



Unit 4: The empty number line

Background

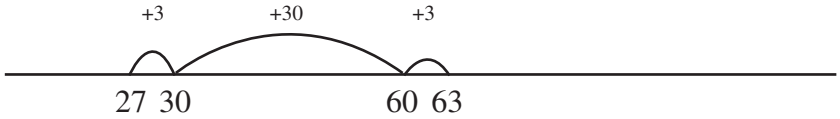
Students usually meet the "number line" as a line already segmented and marked with numerals at regular intervals. This structure builds in "right" and "wrong" positions for numbers. Students also tend to focus on reading someone else's number line rather than on constructing their own.

In contrast, the empty number line can be used flexibly by the student. This has two major advantages. First, the student is in control of what numbers are placed and where. Second, and more important, the stress is on order rather than exact position. For example, if a number line is marked with "0" and "1" at opposite ends, and a student is asked to mark one-half and one-third on the line, the focus is on whether the student places one-half roughly in the centre, and one-third somewhat to the left of one-half, rather than on whether each is placed exactly in its correct position. The emphasis is therefore on order or magnitude of numbers, rather than on measurement.

Specific models

An ideal first model of the number line is the "clothesline", strung across the front of the classroom. Numbers are written on small rectangles of paper and hung with clothes pegs. As this is an easily recognised out-of-school object, and because numbers can easily be placed, moved or removed, it appears to be a very comfortable introduction to the number line for children. Later, a line drawn on the chalkboard or whiteboard can be used by the teacher and the students to assist in communicating solution strategies for computation. For example:

"27 + 36. 27 and 3 is 30, and 30 more makes 60, and 3 more makes 63."



Students can then be encouraged to draw their own number lines for specific questions.

Ordering whole numbers to 100

Activity

***Counting On* reference: Level 0**

Purpose

To order whole numbers up to 100.

Materials needed

Clothesline or string, clothes pegs, rectangles of paper about 10cm x 15cm.

Preparation

Hang the clothesline (string) at about head height across front of the classroom. Peg "0" and "100" at either end of the line. These end points can vary depending on the needs of the students.

Process

Each student writes his or her favourite number (within the given limitations) on a paper rectangle and pegs it on the number line.

Students are asked to give reasons for their favourite number and for its placement. For example, "My favourite number is 57 because that is our house number. It's a bit more than 50 which is half way".

They are also asked to look for any numbers they think are misplaced.

Instructional sequence

- Point to two adjacent hanging numbers (e.g. 57 and 64) and ask for numbers which could be hung between them.
- Hang a blank paper rectangle and ask what number it could represent.
- Ask where a particular number, such as 24 should be placed.
- Individual students can hang a paper rectangle with a number written on the back (provided it cannot be read through the paper) and challenge the class to determine what the number is.
- Individual students can draw a "clothesline" in their books and draw 5 or 10 numbers "hung" on it. The numbers can be chosen by students or provided by the teacher.
- Pairs of students can draw a number line and give each other numbers to place on it, challenging any placement they disagree with.

Extensions

- Give individual students numbers to hang on the line, with just 0 and 100 marked. In particular, give students the multiples of 10 (10, 20, 30...) to hang.
- Give individual students numbers to hang on the line, with just 0 and 100 marked, but have them hang them facing away from the class, so that they do not see the other numbers being hung, and the class cannot see any of the numbers. Then turn each number round and ask if it appears to be in its correct place. If there is disagreement, ask for reasons. In some cases, before turning the number round, ask for suggestions as to what the number might be. Do not necessarily turn the numbers starting from the left, but select numbers at random.
- Ask pairs of students to draw and mark numbers on blank number lines marked from zero to 20, 50, 100, 1000, 10 000, 100 000.

Jumps of ten on the number line

Counting On **reference: Level 0**

Purpose

To help students predict the number of tens in a number.

Materials needed

Blank card, clothesline

Time required

One lesson.

Activity 1

1. Draw a number line on the blackboard with 0 and 100 positioned at the endpoints. Select a multiple of ten and position it on the line, for example:

2. Ask students to determine the number of jumps of ten you would need to make to reach 80.
3. Remove or erase the 80, and repeat with a different multiple of ten.
4. On a number line, position zero and 100. Locate a number with a blank card and ask students to determine which multiple of ten it might represent. Ask students to justify their decision. Repeat with different blank cards.

Double number line

Counting On **framework reference: Level 0, Level 4, Level 5**

Purpose

The double number line can be used to extend understanding of the placement of numbers on a number line.

Materials needed

Photocopiable sheets, paper clips.

Instructions

Each double scale is photocopied onto cardboard, cut out and folded along the middle line. A paper clip is used to estimate the position of a nominated number and the scale is flipped over to check the estimation.

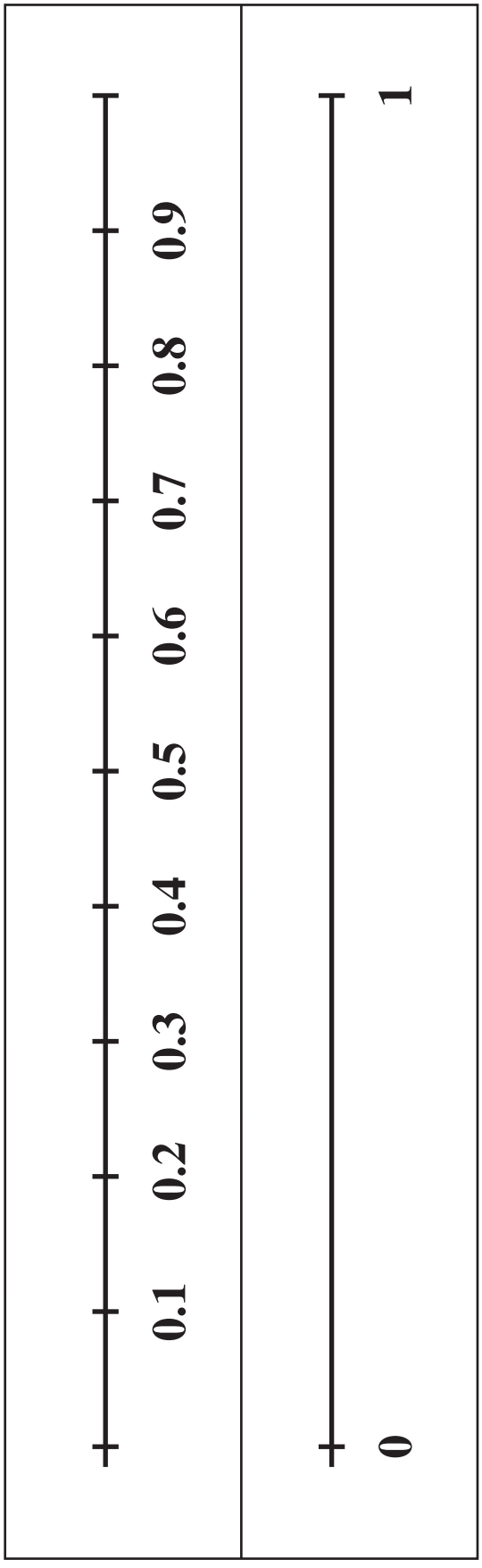
The activity can be an individual, paired or group activity. The side without the additional intervals is displayed and the paper clip used to indicate the nominated position on the scale. Once the estimation is made it can be checked on the other side with the paper clip acting as a dial for both sides.

Discussion of strategies will enable students to improve their understanding and estimations.

The following double number line cards use the same scale on both lines.

Extension

It is possible to use the double number line to illustrate the idea of ratio or proportion by using different scales such as 2:1 or 2:3. This needs to be introduced carefully and goes beyond the focus of these materials.


0
1
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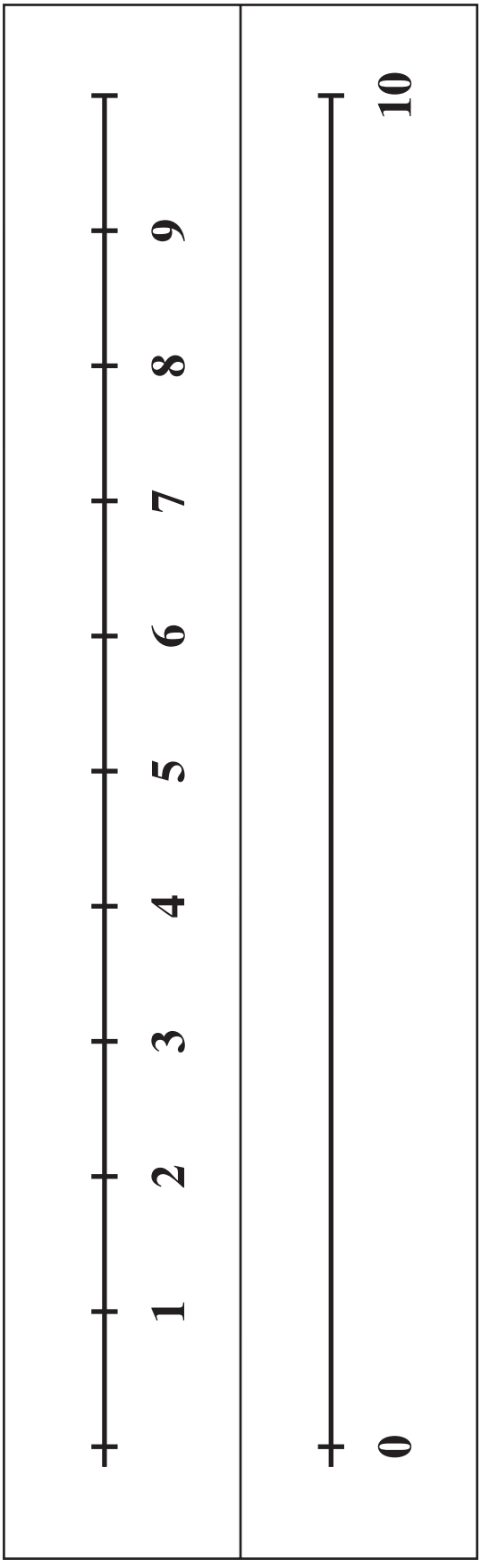
0
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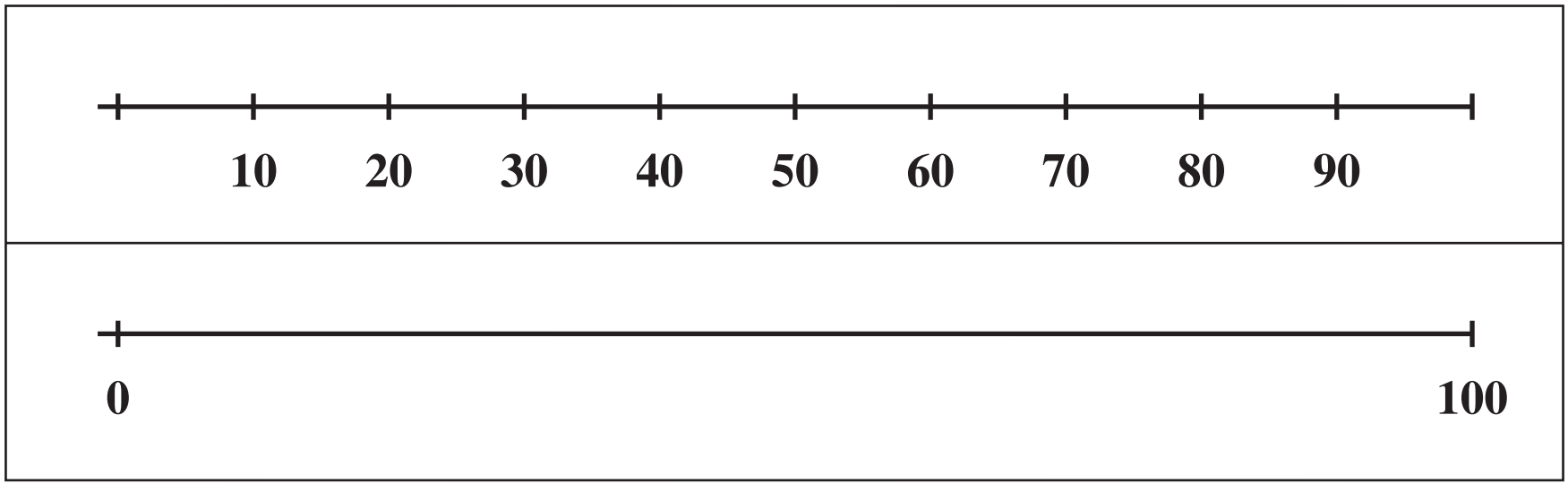
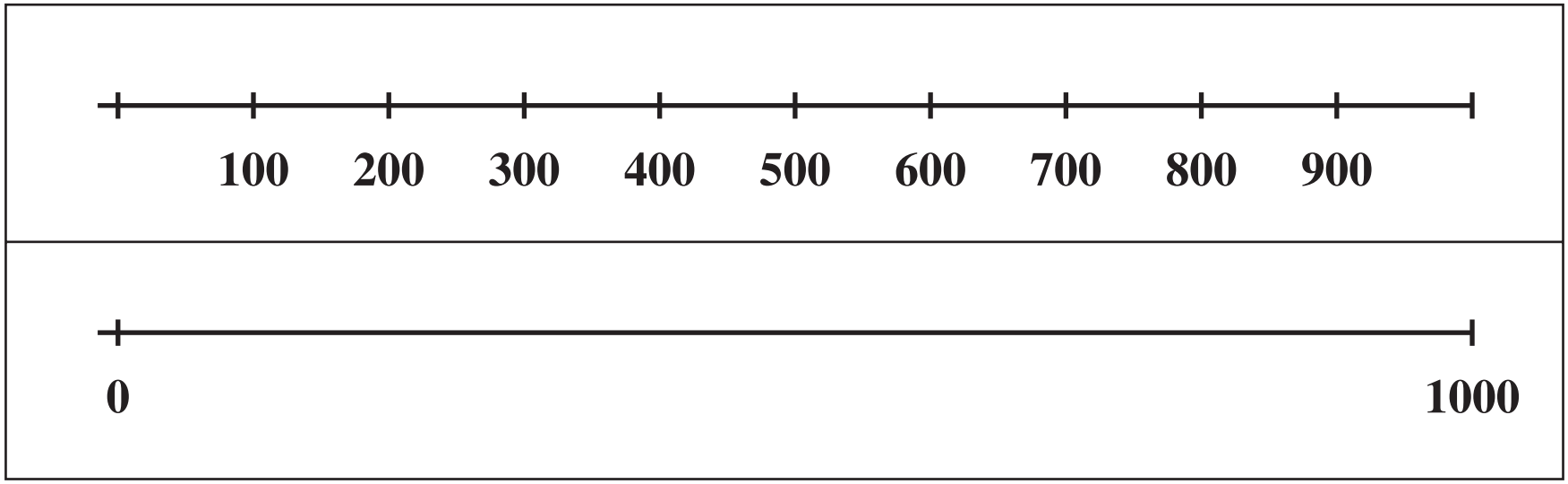
8
9


10 20 30 40 50 60 70 80 90
0 100


100 200 300 400 500 600 700 800 900
0 1000

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0
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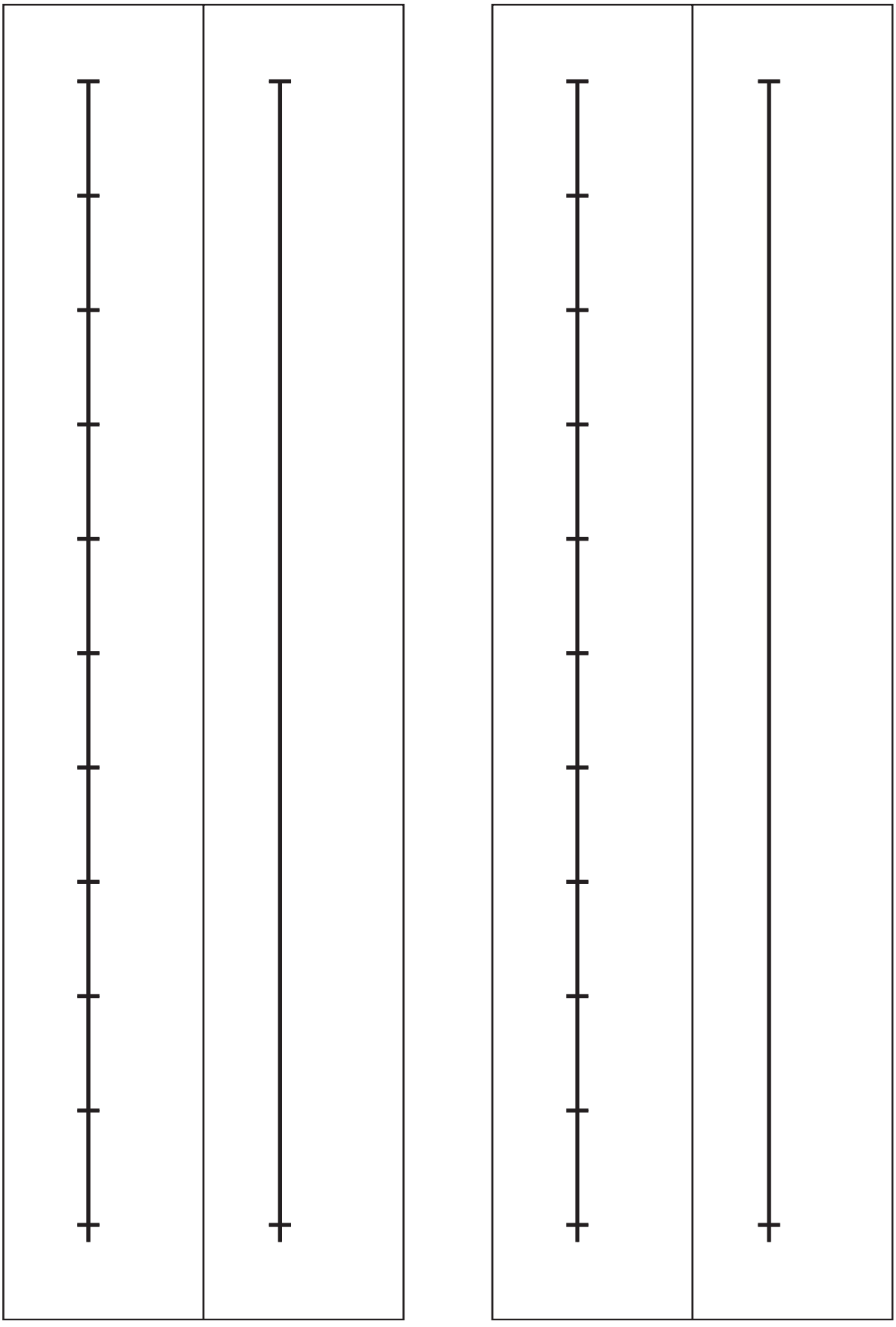


Comparing counting-based and collection-based methods

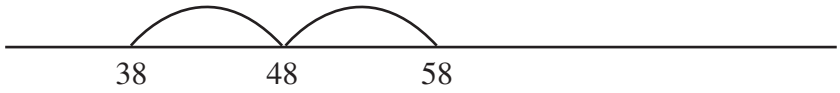
The intent of the lessons that use ten-frames or the empty number line is to help students to progress through the place value levels.

Many tasks can be approached using either a collection–based strategy or via a counting-based strategy.

Task: Display and then screen 38. *Here are thirty-eight dots.*
Display another 20. *If I put out another 20, how many dots are there altogether*?

Counting-based strategy

- When solving this task we could start with 38 and make a ten hop to 48 and another ten hop to 58.
- "Counting" (saying the number words) in multiples of ten is a necessary prerequisite for this strategy. This can be done in conjunction with the hundred chart (e.g. "counting" down the four column: 4, 14, 24, 34 ...)
- Using an empty number line to display or map students' strategies is a very useful device in developing counting-based strategies for adding and subtracting tens and ones.



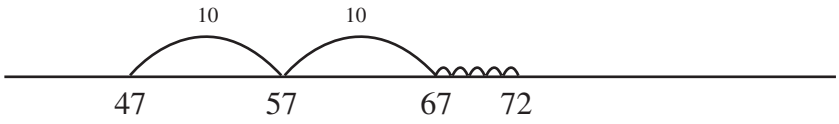
Collection-based strategy

- Using a collection–based strategy in the example above (38 and 20), the teacher would have students find how many tens there were. Students could initially construct the numbers with ten-frames and collect the five tens and then see that there are five tens and eight ones which is 58.
- After making models of these additions (or subtractions), the teacher would progress to giving verbal tasks without the use of tens material.

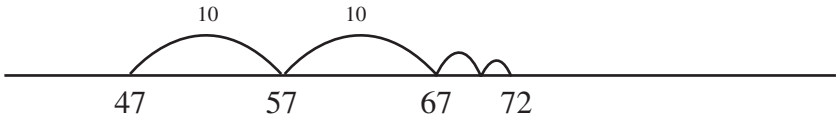
Task: Display then screen 47. *Here are forty-seven dots.*
Display another 25. *If I put out another twenty-five, how many dots are there altogether*?

Counting–based strategy

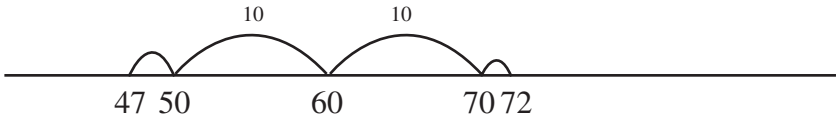
- Using the empty number line, students could start at 47 and use one of the following strategies:
  (a) Start at 47. Make a 10 jump to 57. Make a 10 jump to 67. Make five single hops to 72.



  (b) Start at 47. Make two 10 jumps: 57, 67. Make a 3 hop to 70. Make a 2 hop to 72. This strategy involves partitioning the five to make the next ten.



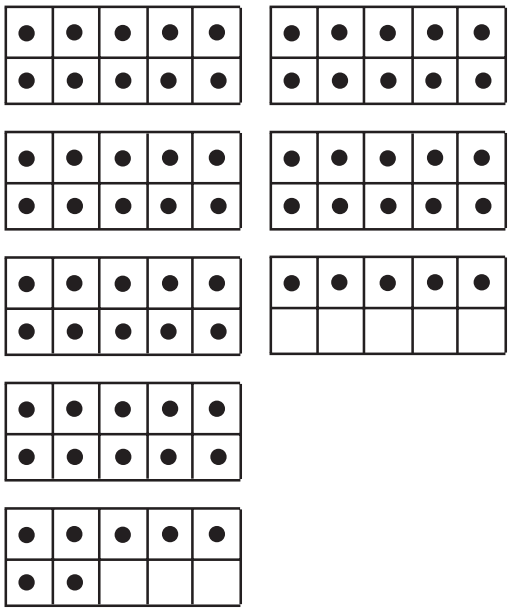
  (c) Start at 47. Make a 3 hop to 50. Make two 10 jumps: 60, 70. Make a 2 hop to 72. This strategy involves initially partitioning the five to make the next ten.



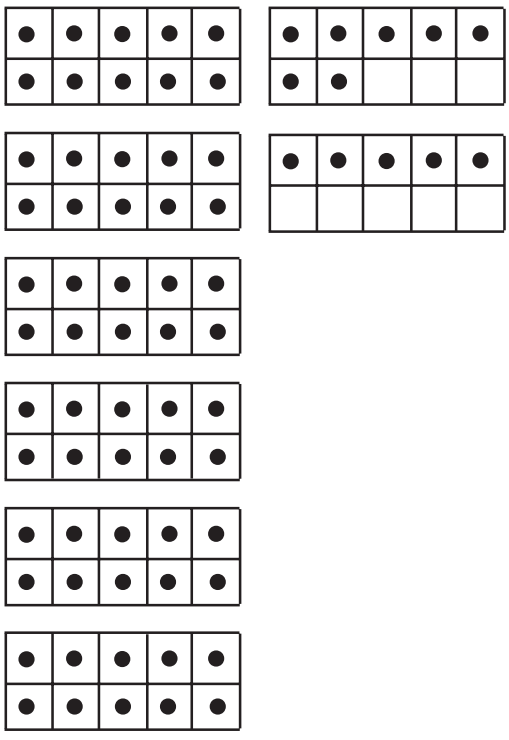
Collection–based strategy

(a) Students make a model using tens–frames

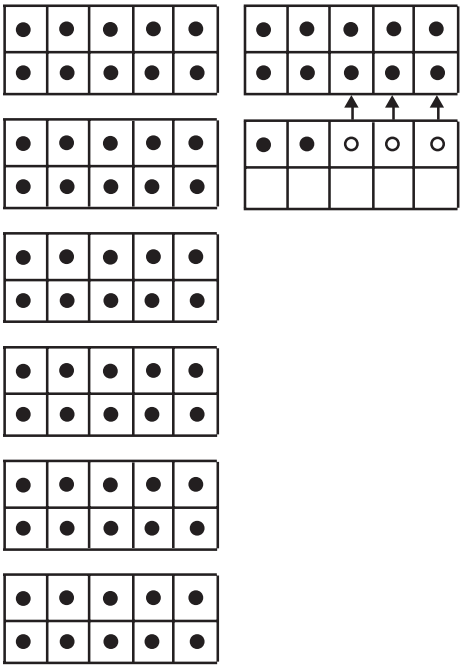
Students make models of 47 and 25.



(b) The tens are grouped.



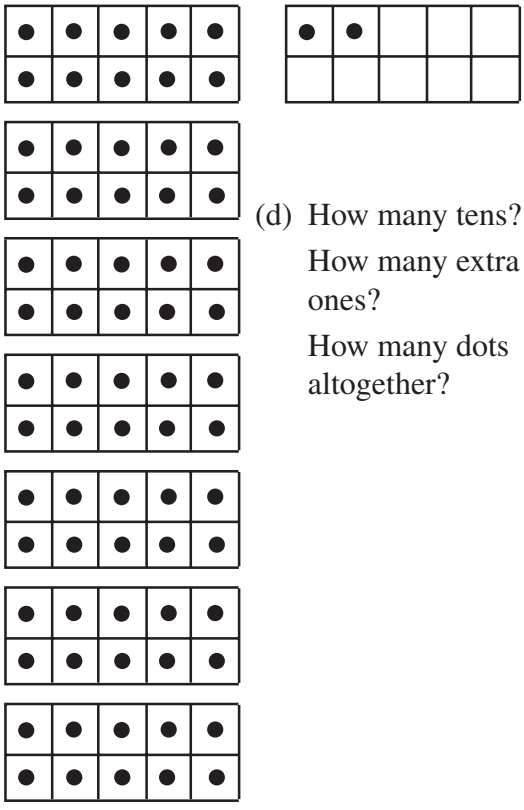
(c) Can we make another ten?



(d) How many tens?

How many extra ones?

How many dots altogether?



Notes

Unit 5: Hundreds chart

Purpose

To strengthen the understanding of standard and non-standard partitioning of two-digit numbers. For example, 36 can be seen as 3 tens + 6 (standard partitioning), and 2 tens + 16 (non-standard partitioning). These partitionings can be useful when adding and subtracting two-digit numbers.

Rationale

One common and effective strategy for mentally adding and subtracting two-digit numbers involves starting with one number, then adding first the tens digit, and then the units digit of the second number. For example, 37 + 25 becomes 37 + 20 = 57 and + 5 = 62. A subtraction example would be 75 - 42 becomes 75 - 40 = 35 and - 2 = 33.

Prerequisite skills

1. Splitting a two-digit number into tens and units. This means being aware that you can add or subtract the tens and units separately.
2. Adding or subtracting a multiple of ten. This requires understanding that operating on the tens digit does not affect the units digit. Moving between rows on the hundred chart increases by ten and between columns increases by one. This chart is useful since the multiples of ten are at the end of the row enabling easy positioning of numbers. The 1 – 100 board is useful for addition and subtraction. The board is arranged with the multiples of ten in the last column. This is helpful when bridging numbers to ten.

Locating a number

Stage 3

Activity 1

Counting on framework reference: Level 0

Materials needed

One 0 – 99 chart and three (preferably transparent) counters per student; demonstration chart (optional) for the teacher.

Time required

One lesson.

Instructional sequence

1. Check that students can locate numbers on the chart. “Place a counter on 37”. “Place a counter on 4”. “Place a counter on 93”. Ask: “How did you find 37? How did you know where to look for 93?” Establish that the tens move down the chart, while the units move from left to right.

2. "Place a counter on zero. What instructions would you give me to move it to 24?" Give students time to discuss this in pairs and then ask for suggestions. Establish that one way is: "Move down two rows and move four squares to the right".
3. Students now work in pairs giving each other similar challenges, for example: "Place your counter on zero. How would you move it to 81?", or "I've placed my counter on zero, and moved it down 4 and 8 to the right. What number is my counter on now?"
4. Give pairs of students this challenge: "One student places a counter on zero and then closes his or her eyes. The other student gives a number, for example, 65. The first student attempts to move the counter onto 65 while keeping eyes closed, using the strategy learned above: moving the counter an estimated six rows down and 5 columns to the right. Score a point for each time you land on the number." Students now play this in pairs.

Tracks

Stage 3

Activity 2

Counting On **framework reference: Level 0**

Purpose

To assist adding and subtracting by tens and ones.

Materials needed

1–100 chart, photocopiable sheets.

Instructional sequence

Begin by modelling the activity on an OHP. Ensure that students understand how to move around the hundreds chart. That is, to add 10 you move down the column of numbers and to subtract 1 you move back one space. Ask students what calculation is made by using diagonals. Possible answers include: + 11 (or + 10 + 1), - 11, + 9, - 9

Fill in your starting number of say, 24 and then draw three directional arrows ↓↓→ . Have a student find out what number you would finish on.

Have students work in pairs. The first student enters a number and completes the arrows, e.g. 43 ↓↓→ and the partner then uses the hundred chart to determine the number that follows the direction, in this case 64. Players take turns to test each other. The proforma on p. 72 provides instruction for five directional arrows.

Extensions

- Students can record the number sequence, e.g. 43 + 10 + 10 + 1 = 64
- Students could complete the activity using a blank hundreds chart.
- Use a numbered 1–100 chart and a blank die marked with compass direction and an arrow, e.g. N, S, E, W, NW, SE. Students need a counter each to keep track. Both players begin on the number 45. Winner could be first to reach 1 or 100.

Tracks

Name: _____

	Number	Arrows			Number
1.					
	Number	Arrows			Number
2.					
	Number	Arrows			Number
3.					
	Number	Arrows			Number
4.					
	Number	Arrows			Number
5.					

Tracks

Name: _____

	Number			Arrows			Number
1.							
2.							
3.							
4.							
5.							

Hundreds chart jigsaw

Activity 3

Counting On **framework reference: Level 0, Level 1**

Purpose

To strengthen students' understandings of adding and subtracting in tens and ones.

Materials needed

Copies of worksheets and scissors.

For extension, prepare jigsaw onto light card and copy blank grid.

Time required

One lesson.

Instructional sequence

1. Provide pairs of students with the page *Missing numbers on the hundreds chart*. Students fill in the missing numbers by either adding or subtracting by tens or ones.

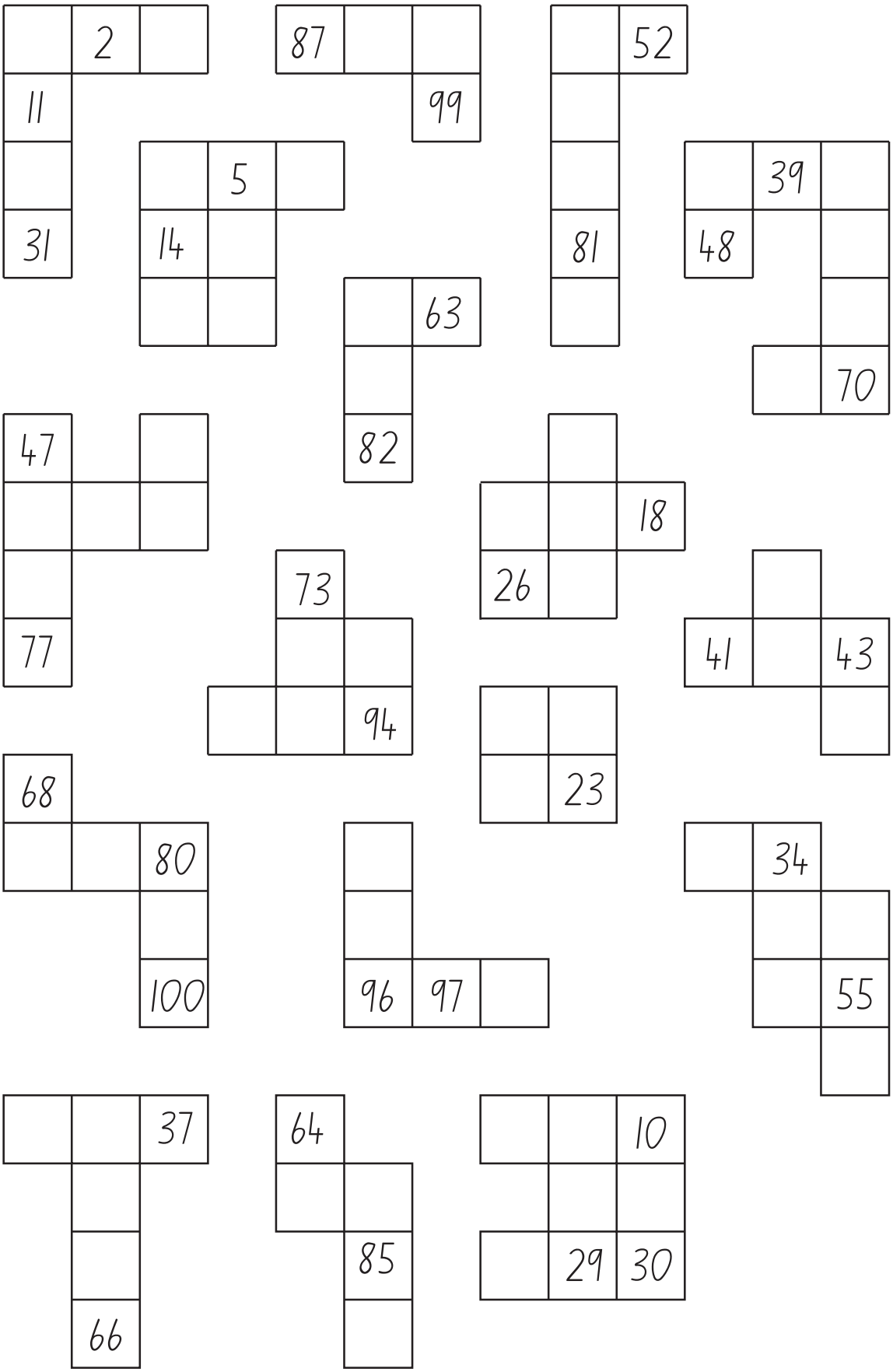
2. When completed, students cut out the pieces. The pieces will fit together to form a hundreds chart.

Extension

Prepare hundreds chart jigsaws on page 75 by photocopying onto light card and cut along the heavy lines.

Students do not write the missing numbers but complete the jigsaw using the blank grid on page 76 as a base.

Missing numbers on the hundreds chart



Hundreds chart jigsaw

				5					10
	12							19	
21			24		26				
	32					37			
			44					49	
		53			56		58		
61			64						
			74				78		
81							88		90
		93			96				100

Adding two 2-digit numbers (when the total of the units digits is 10 or less)

Activity 4

***Counting On* framework reference: Level 1, Level 2**

Materials needed

One 1–100 chart and 3 (preferably transparent) counters per student; demonstration board (optional) for the teacher. Enlarge proforma to fit counters.

Time required

One lesson.

Instructional sequence

1. Introduce the 1–100 chart to the students and ask in what ways it is different from, or the same as the 0–99 chart. Explain that locating a number by moving down (for tens) and then across (for units) is essentially the same as using a 0–99 chart, except that the first number on the 1–100 chart is a one.

2. Place a counter on 34. "How could I move this counter forward 23 squares?" Expect some students to suggest moving it on one square at a time. Ask if students can think of a quicker or more efficient way of moving on 23 squares. Following steps 1–4 in Activity 1, it is likely that some students will suggest moving the counter "2 squares down and 3 squares to the right." Before making these moves ask: "What square will I land on?" Establish that these moves have the effect of adding 34 and 23, producing a sum of 57.

3. Students now practise this strategy in pairs, one making the moves and the other giving the addends and checking the moves. Give students a series of additions to perform using this strategy. (The teacher needs to provide the additions to ensure that none of the units total more than 10, for example: 12 + 13, 50 + 30, 23 + 35, 32 + 53, 26 + 30, 41 + 49, 53 + 27, 44 + 5, 71 + 18, 13 + 83).

4. If students find this relatively easy, ask them to find as many pairs of numbers as possible which have a sum of 100 (by placing the counter on any number and then finding what moves they have to make to land the counter on 100).

Tens and ones: The jump method

Activity 5

***Counting On* framework reference: Level 1, Level 2, Level 3 (extension)**

Purpose

To help students understand the *jump* method of addition and subtraction.

Materials needed

Overhead projector, transparency of 1–100 chart, cardboard sheet with "window" cut out to reveal one number. Enlarge proforma to fit counters.

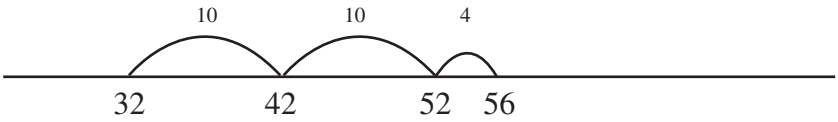
Time required

One to two lessons.

Instructional sequence

1. The teacher shows the uncovered 1–100 chart on the overhead projector. Students are invited to demonstrate ways of moving a transparent coloured counter from say, 32 to 56, by ones as 24 moves or by tens and ones ("down" 2 and "across" 4) as 6 moves. This provides a powerful visual representation that one method may be much more efficient than another.

2. The teacher encourages discussion on the interpretation of these strategies into symbols: 32 + 24 = 56. The teacher may also ask a student to demonstrate the strategies on an empty number line.



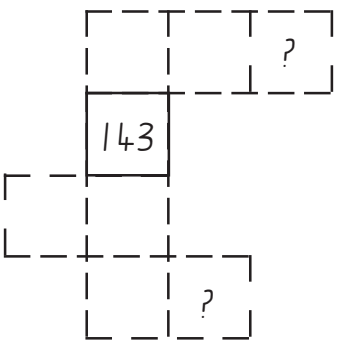
3. The teacher covers the 1–100 chart so that only one number (say) 43 is visible through the "window" card that covers all other numbers. Questions can then take one of two forms: "*Tell me how to move to 67*." or "*Tell me how to add 23*." Questions involving subtraction may also be asked and translated onto the blank number line.

Extension

The teacher uses a 101–200 chart. As the "window" is moved students should notice when tens are added (i.e. downward movements), the hundreds and units digits remain unchanged.

Variations

The teacher uses the 101–200 chart transparency and "window" to show say, 143 on the chalkboard. The teacher then draws "extra" squares onto the chalkboard. Students are asked to work out the numbers in the empty squares.



Subtracting two 2-digit numbers (when the units digit of the second number is less than or equal to that of the first number).

Activity 6

***Counting On* framework reference: Level 1**

Materials needed

One 1–100 chart and 3 (preferably transparent) counters per student. Demonstration chart (optional) for the teacher. Enlarge proforma to fit counters.

Time required

One lesson.

Instructional sequence

This sequence uses exactly the same strategy as used in Activity 4, except that for subtraction, the counter will move up and to the left.

1. Place a counter on 34. "How could I move this counter back 23 squares?" Expect some students still to suggest moving it back one square at a time. Ask if students can think of a quicker or more efficient way of moving back 23 squares. Students might suggest that you can "move 2 squares up and 3 squares to the left". Before making these moves ask: "What square will I land on?" Establish that these moves have the effect of subtracting 23 from 34, producing a result of 11.

2. Students now practise this strategy in pairs, one making the moves and the other giving the numbers and checking the moves. Give students a series of subtractions to perform using this strategy. (The teacher needs to provide the subtractions to ensure that the units digit of the second number is less than or equal to that of the first number, for example: 25 - 13, 80 - 30, 58 - 35; 85 - 53, 56 - 30, 90 - 49, 80 - 27, 49 - 5, 89 - 18, 96 - 83).

3. If students find this relatively easy, ask them to find as many pairs of numbers as possible which have a difference of 51.

Adding two 2-digit numbers (when the units digits total more than 10).

Activity 7

***Counting On* framework reference: Level 1**

Materials needed

One 1–100 chart and three (preferably transparent) counters per student; demonstration chart (optional) for teacher. Enlarge proforma to fit counters.

Time required

One lesson.

Instructional sequence

There are three possible strategies that students should be encouraged to use with the 1–100 chart. The following instructional sequence describes all three strategies.

1. Place a counter on 48. "How could I move this counter forward 29 squares?" Following the previous activities, it is likely that some students will suggest that you can "move 2 squares down and 9 squares to the right." Before making these moves ask: "What square will I land on?" Establish that in this case it is impossible to move more than 2 squares to the right. Ask for suggestions. Most students are likely to suggest that you "move to the start of the next row and keep counting". Some students may do a mental calculation and say that you know it will land on 77 because "from 68, using 2 from the 9 makes 70, and there are 7 more left". Other students may suggest that "you can move down one more row (onto 80) and then move back (to the left) 1 square".

   Establish that the first strategy is equivalent to adding 20 in tens and then adding the 9 mainly by counting on in ones.

   The second strategy is equivalent to "bridging". This is done by adding 20 to make 68, then 2 (to make 70) and then 7.

   The third strategy is equivalent to adding 30 and then subtracting 1. Some students will find this strategy less easy to assimilate. Encourage students to use and compare all the strategies, but make it clear that the important thing is to feel comfortable with at least one of the strategies.

2. Students now practise a strategy in pairs, one making the moves and the other giving the addends and checking the moves. Give students a series of additions to perform using their preferred strategy. (The teacher needs to provide the additions to ensure that the units total more than 10, for example: 13 + 9, 8 + 24, 26 + 25, 37 + 25, 45 + 28, 39 + 52, 35 + 47, 63 + 19, 60 + 26, 39 + 59).

3. If students find this simple, they can play this game in pairs. Each student writes down any number less than 50. They then simultaneously reveal their number to each other: the first to produce their sum (either by using the 1–100 chart or mentally) wins a point.

Subtracting two 2-digit numbers (when the units digit of the second number is greater than that of the first number).

Activity 8

***Counting On* framework reference: Level 1, Level 2**

Materials needed

One 1–100 chart and three (preferably transparent) counters per student, demonstration chart (optional) for the teacher. Enlarge proforma to fit counters.

Time required

One lesson.

Instructional sequence

This sequence uses exactly the same strategy as used in Activity 7, except that for subtraction the counter will move up and to the left.

Again, there are three possible strategies that students should be encouraged to use. The following instructional sequence describes all three strategies.

1. Place a counter on 77. "How could I move this counter back 29 squares?" Following earlier activities, it is likely that some students will suggest that you can "move 2 squares up and 9 squares to the right." Before making these moves, ask: "What square will I land on?" Establish that in this case you can move two squares up, to 57, but that it is impossible to move more than 6 squares to the right. Ask for suggestions. Most students are likely to suggest that you "move to the end of the next row and keep counting back". Some students may do a mental calculation and say that you know now it will land on 48 because "from 57, 7 from the 9 made 50, and there are 2 more left, making 48". Other students may suggest that you can move up one more row (onto 47) and then move on (to the right) 1 square to 48".

   Establish that the first strategy is equivalent to subtracting 20 in tens and then subtracting the 9 mainly by counting back in ones.

   The second strategy is equivalent to "bridging". This is done by subtracting 20 to make 57, then 7 (to make 50) and then 2.

   The third strategy is equivalent to subtracting 30 and then adding 1. Some students will find this strategy less easy to assimilate. Encourage students to use and compare all the strategies, but make it clear that the important thing is to feel comfortable with at least one of the strategies.

2. Students now practise a strategy in pairs, one making the moves and the other giving the addends and checking the moves. Give students a series of subtractions to perform using this strategy. (The teacher needs to provide the subtractions to ensure that the units digit of the second number is greater than that of the first number, for example: 22 - 9, 32 - 24, 51 - 25, 62 - 25, 73 - 28, 91 - 52, 82 - 47, 82 - 19, 86 - 26, 98 - 59).

3. If students find this simple, they can play this game in pairs. One student writes down any number greater than 50, while the other student writes down any number less than 50. They then simultaneously reveal their number to each other. The first to produce their difference (either by using the 1–100 chart or mentally) wins a point.

1–100 Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0–99 Chart

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Shapes on the 0–99 chart and the 1–100 chart

Purpose

To reinforce properties of simple shapes (square, rectangle, parallelogram, isosceles triangle); look for simple relationships between numbers; reinforce tens and units aspects of numbers; make and test hypotheses.

Activity 1

Making two-by-two squares.

***Counting On* framework reference: Level 2, Level 3**

Materials needed

One 0–99 or 1–100 chart (both are equally good for this activity) and four (preferably transparent) counters per student; demonstration chart (optional) for teachers.

Time required

One or two lessons.

Instructional sequence

1. The teacher explains that a "two-by-two" square is any group of four numbers in a square shape, such as 45, 46, 55, 56. Students place their four counters on the numbers in a two by two square anywhere on the 0–99 or 1–100 chart. The numbers are recorded and the four numbers covered.

2. The teacher records on the board some of the sets of four numbers recorded by students (for example: 23, 24, 33, 34; 47, 48, 57, 58) and asks the students to describe any patterns or relationships they see in the sets of four numbers. For example, there are always two pairs of consecutive numbers; the two bigger numbers are 10 more than the two smaller numbers.

3. The teacher asks for reasons for these relationships to ensure that students can see and explain these in terms of the structure of the board. For example, two numbers next to each other on a line are like counting numbers. Any number is 10 more than the number above it.

4. Students now work in pairs to write down (without looking at the board) sets of four numbers they think will form a 2 x 2 square. Each student then checks the other's sets by inspection and by placing counters on the board.

5. The teacher then gives students ten sets of four numbers and challenges them (individually or working in pairs) to decide which sets would form a 2 x 2 square, first by inspecting the numbers and then checking by placing them on the board.

Extension

Challenge students to develop ways to find the total of a 2 x 2 square.

Activity 2

Making three-by-three squares.

Counting On **framework reference: Level 2, Level 3**

Materials needed

One 0–99 or 1–100 chart (both are equally good for this activity) and four (preferably transparent) counters per student; demonstration chart (optional) for the teacher.

Time required

One or two lessons.

Instructional sequence

This activity follows the same steps as Activity 1, but students are now placing counters at the corners of 3 x 3 squares.

1. The teacher asks students to place their four counters on these four squares: 23, 25, 43, 45 and asks the students to describe the arrangement. For example, the counters are at the corners of a square. It's a larger square. The teacher invites students to suggest a description of this size of square and establishes that it is called a "three-by-three" square.

2. Students place their four counters on the squares at the corners of a three-by-three square anywhere on the 0–99 or 1–100 board and record the four numbers covered.

3. The teacher records on the board some of the sets of four numbers recorded by students (for example: 13, 15, 33, 35; 47, 49, 67, 69) and asks the students to describe any patterns or relationships they see in the sets of four numbers. For example, there are always two pairs of numbers which differ by two; the two bigger numbers are 20 more than the two smaller numbers.

4. The teacher asks for reasons for these relationships to ensure that students see and explain these in terms of the structure of the board. For example, if you move two squares to the right you are counting on two or adding two. Any number is 20 more than the number two squares above it.

5. Students now work in pairs to write down (without looking at the board) sets of four numbers that they think will form a 3 x 3 square. Each student checks the other's sets by inspection and by placing counters on the board.

6. The teacher then gives students ten sets of four numbers and challenges them (individually or working in pairs) to decide which sets would form a 3 x 3 square, first by inspecting the numbers and then checking by placing them on the board.

Extension

Challenge students to develop strategies to find the total of a 3 x 3 square.

Activity 3

Making larger squares.

Counting On **framework reference: Level 2, Level 3**

Materials needed

One 0–99 or 1–100 chart (both are equally good for this activity) and four (preferably transparent) counters per student, demonstration chart (optional) for the teacher.

Time required

One or two lessons.

Instructional sequence

1. The teacher asks individual students to place their counters at the corners of a "four-by-four" square (without explaining the term), and to record the four numbers. Students compare their squares and numbers with a partner, and look for patterns in their numbers and in those of their partners.

2. The teacher now issues a challenge: "Make squares of different sizes which are large and small. Record the four numbers and the size of the square and look for relationships between the four numbers and the size of the square. In 30 minutes (or whatever time is appropriate) I will give you 10 (or 15 or 20) sets of four numbers for you to determine:
   (a) whether they form a square, and
   (b) if so, how large a square. You won't be able to use your 100 boards to help you."

3. Students work in pairs to make squares of different sizes and to explore relationships.

4. Challenge students to develop strategies to find the total of a 4 x 4 square.

Activity 4

Making rectangles.

Counting On **framework reference: Level 2, Level 3**

Materials needed

One 0–99 or 1–100 chart (both are equally good for this activity) and four (preferably transparent) counters per student; demonstration chart (optional) for the teacher.

Time required

One or two lessons.

Instructional sequence

1. The teacher asks students to place their four counters on these four squares: 23, 27, 43, 47 and asks the students to describe the arrangement. For example, the counters are at the corners of a rectangle. The teacher invites students to suggest a description of this rectangle and establishes that it is called a "five-by-three" rectangle.

2. The activity can now follow the general sequence of Activities 2 and 3, depending on the students' understanding and interest. The most general description of the relationship which can be expected is that there are two pairs of numbers which each differ by the same amount (between 1 and 9) and two pairs which differ by the same multiple of ten (between 10 and 90), and that the size of the resulting rectangle is "one more than the difference between the units digits" by "one more than the difference between the tens digits".

3. Students calculate the total of the 5 x 3 rectangle.

Activity 5

Making parallelograms.

***Counting On* framework reference: Level 2, Level 3**

Materials needed

One 0–99 or 1–100 chart (both are equally good for this activity) and four (preferably transparent) counters per student, demonstration chart (optional) for the teacher.

Time required

One or two lessons.

The relationships here are less obvious and require more mental computation ability, for example to notice that, in the example given below, the first and third numbers and the second and fourth numbers both differ by 22.

Instructional sequence

1. The teacher gets students to place their four counters on these four squares: 23, 27, 45, 49 and asks the students to describe the arrangement. For example, the counters are at the corners of a parallelogram. Students are then invited to form their own parallelograms and look for relationships between the four numbers. It is important for simplicity to make the rule that one pair of opposite sides must be horizontal, i.e. each lie along one row of the board. The "size" of the parallelogram is more difficult to describe and may be omitted.

2. The activity can now follow the general sequence of Activities 2 and 3, depending on the students' understanding and interest. The most general description of the relationship which can be expected is that there are two pairs of numbers which each differ by the same amount (between 1 and 8) and two pairs which differ by the same (usually two-digit) number.

Activity 6

Making isosceles triangles.

Counting On **framework reference: Level 2, Level 3**

Materials needed

One 0–99 or 1–100 chart (both are equally good for this activity) and three (preferably transparent) counters per student; demonstration chart (optional) for the teacher.

Time required

One lesson.

This activity follows the general pattern of Activity 5 and may be used before or instead of it. It is wise to insist that the base of the triangle be horizontal. One example of an isosceles triangle is 34, 63, 65.

Unit 6: Activities for place value

First to 100

Purpose

To help students increment by tens and units to 100 using strategies other than counting by one.

Materials needed

A set of cards (A) for each group of students, or a die.

A set of cards (B) for each group of students.

Time required

30 minutes.

Stage 3

Activity

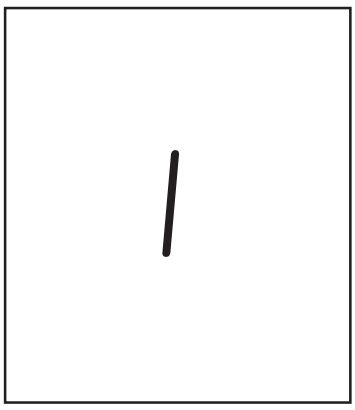
Counting On **framework reference: Level 1**

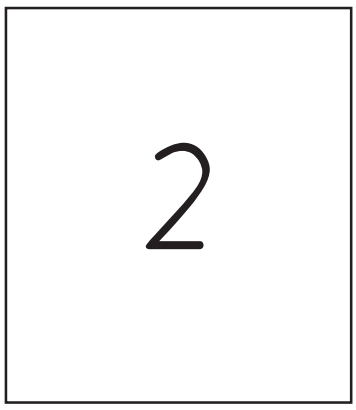
1. In pairs, students take a card from Cards A and places it on the desk with the number facing up. Each student records his or her number clearly at the top of a blank page. The person with the highest number goes first. Alternatively a die could be thrown.

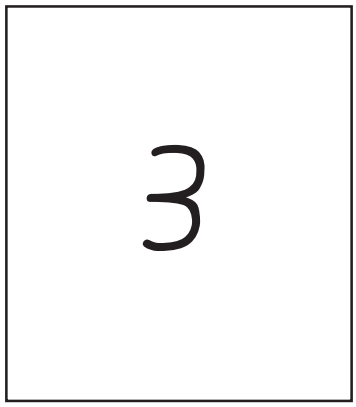
2. Students in turn take a card from Cards B and mentally add their two numbers together. The process used to add the numbers is said aloud by the student solving the addition problem. This number is recorded on the page below the original number. The other student checks the addition before continuing.

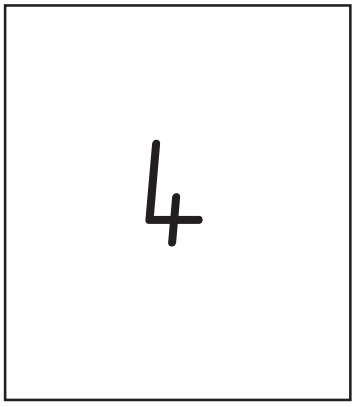
3. The first student to reach 100 wins. Adjust the final total according to the needs of the students.

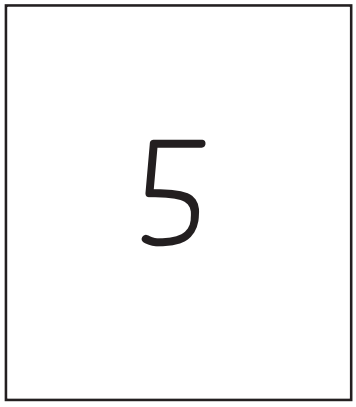
Cards (A)

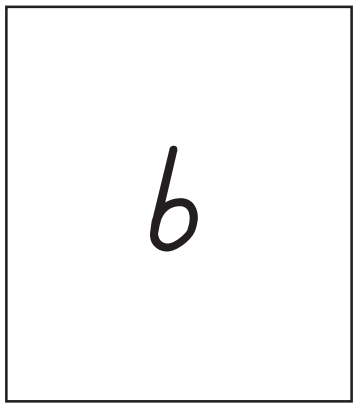












Cards (B)

ADD 1	ADD 1	ADD 1
ADD 1	ADD 1	ADD 1
ADD 10	ADD 10	ADD 10
ADD 10	ADD 10	ADD 10
ADD 20	ADD 20	ADD 20
ADD 30	ADD 30	ADD 30

SUBTRACT 1	SUBTRACT 1	SUBTRACT 1
SUBTRACT 1	SUBTRACT 1	SUBTRACT 1
SUBTRACT 10	SUBTRACT 10	SUBTRACT 10
SUBTRACT 10	SUBTRACT 10	SUBTRACT 10
SUBTRACT 20	SUBTRACT 20	SUBTRACT 20

Add to 100

Purpose

To support the addition of 2-digit numbers to 100 using strategies other than counting by ones.

Materials needed

The grid below.

Time required

15 minutes.

Instructional sequence

Activity 1

***Counting On* framework reference: Level 1, Level 2**

1. Individually, or in pairs, students find all groups of two or three numbers, side by side, across or down, which can be added together to make 100.
2. These are marked or circled on the grid.
3. Students record these in their books. Encourage students to use a range of strategies to add the numbers. This might include adding the tens first then the units; adding the ten onto the first number, then the units. Discourage counting on by ones.

30	70	22	48	30	79	10
33	92	8	9	55	21	90
33	20	80	1	45	22	78
34	18	91	90	50	25	25
64	52	9	71	8	21	10
36	30	11	44	45	43	55
29	40	31	4	96	57	35

Make 100

Purpose

To help students to group tens and ones and add tens and ones.

Counting On **framework reference: Level 1, Level 2**

Materials needed

One die for the teacher (1–6 or 0–9); scoring sheets: one per student.

Time required

Two lessons.

Rules

1. The aim is to score 100 or as close as possible without 'busting' (passing 100).

2. The teacher rolls the die and announces the number. Students may choose to multiply that number by 10 or score it at face value, e.g. 2 may be scored as 2 or 20. Once a decision has been made it cannot be changed.

3. The die is rolled again. If the number is (say) 4, students decide to score this as 4 or 40 and record it, completing the progressive total.

4. This continues until 9 rolls have been completed. Note: All rolls must be used.

5. The student who scores 100 or the number closest to (but below) 100 wins.

Variations

(a) Use a 1–6 die or a 0–9 die. Ask students how they will vary their strategies if you change from a 1–6 to a 0–9 die.

(b) Set a different target.
   (i) Target = 200 "How will you vary your strategies from the original game?" (Students should realise that they will need to multiply by 10 more often.)
   (ii) Target = 1000 and you may multiply by 100 once and once only during the game.

(c) Allow addition or subtraction of each number rolled.

Sample game

Score	Total
2	2
30	32
6	38
10	48
3	51
20	71
4	75
5	80
4	84
	84

Questioning

1. After two rolls the teacher asks students to name all possible scores. (In the sample game above the possible scores are 5, 23, 32, and 50.) Note: If the first two numbers rolled are the same, there are only three possible scores, e.g. rolls of 3, 3 produce possible scores of 6, 33, 60.

2. When there are two rolls to go, ask students to stand if it is still possible for them to score exactly 100. Ask their scores and check as a whole class activity.

3. When there are two rolls to go, ask students to announce their scores and say what they would like the next two rolls to be, e.g. a student with a score of 88 could score exactly 100 with 2 sixes. The same result could be achieved with a 1 and a 2 (12).

Extension

Refer to activities “Make 10” and “Make 1” on p. 197.

Make 100

Score	Total

Score	Total

Score	Total

Score	Total

Score	Total

Score	Total

Flip 4 and add

Counting On **framework reference: Level 1**

Purpose

To encourage students to use a mental computation approach to add two 2-digit numbers.

Materials needed

Two sets of numeral cards 1–9 for each pair or group of students.

Pencil and paper or work book for recording.

Time required

One lesson.

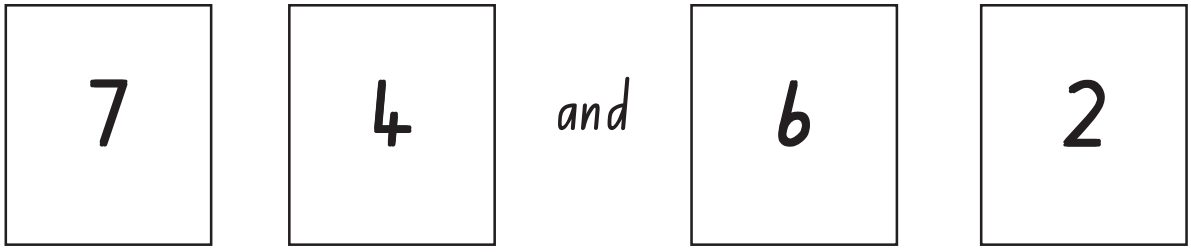
Instructional sequence

Activity 1

1. In pairs or groups of 3 or 4, students place the numeral cards face down on the desk.

2. Each student takes a turn to pick up one numeral card, until all players have four cards each.

3. Students make two numbers with their cards. Each number must have two digits, for example:



4. Students add their two numbers mentally by adding the tens digits first and then the units. For example, 70 and 60 is 130 and 4 is 134 and 2 is 136, or, 70 and 60 is 130, 4 and 2 is 6, 130 and 6 is 136. The sum and the strategy used are then recorded in their workbook.

5. Students can play so that 1 point is scored for the highest addition obtained.

Variation

Make the two numbers and then subtract.

Flip 4 and add cards

1	2	3	4
1	2	3	4
5	6	7	8
5	6	7	8
9	9		

Flip 2 and subtract

Counting On **framework reference: Level 1**

Purpose

To encourage students to use a mental computation approach to subtract two numbers.

Materials needed

A set of cards from 1 to 30.

Pencil and paper or work book for recording.

Time required

10 minutes.

Activity 1

1. In pairs students flip two cards and find the difference between the two numbers shown.

2. Adjust the range of cards used according to the needs of the students. It might be useful initially to limit the numbers that will be used for the subtractions to 1 to 20.

Flip and move cards: Flip 2 and subtract

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20

Flip and move cards: Flip 2 and subtract

21	22	23	24
25	26	27	28
29	30		

Flip and move

Counting On **framework reference: Level 0**

Purpose

To strengthen students' flexibility with the positioning of numbers.

Materials needed

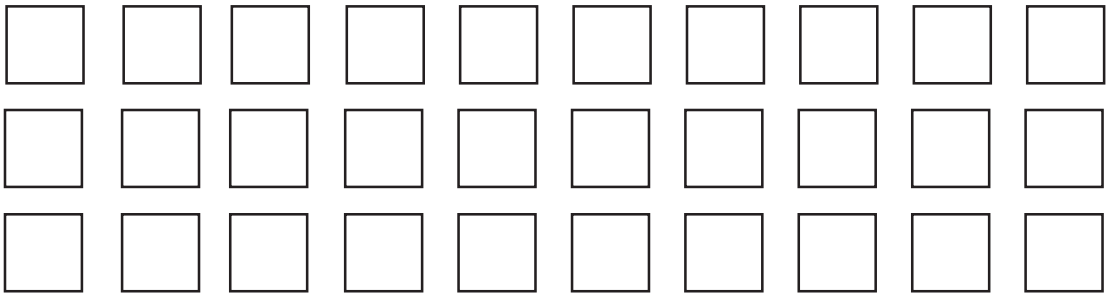
A set of numeral cards from 1 to 30.

Time required

15 minutes.

Activity 1

1. In pairs, students randomly place the 30 numeral cards in three rows of 10 cards as shown.



2. One student flips one of the cards and decides where it should be placed. For example, if a student flips the card 22, he or she should say that it belongs in the third row, and count, 21, 22.

3. The card that is currently in the 22 position is picked up by the next student. This card is turned over and moved to its correct position.

Extensions

- Use numbers 71–100.
- Use 1 to 30 but have students arrange numbers in reverse order. 30–1.
- Use number cards 1–100

Make some numbers

Counting On **framework reference: Level 2**

Purpose

To help students order numbers with 3 or 4 digits.

Materials needed

One set of cards with the digits 0–9 for each student.

Time required

One lesson.

Instructional sequence

Activity 1

1. In pairs, students deal themselves three single-digit cards.

2. Students write down the six possible numbers that can be generated using those three digits. If a zero is selected, then include the numbers with zero in the hundreds place.

3. Students then each order their six numbers from smallest to largest and record how many groups of ten could be formed with the smallest number. Pairs of students swap their solutions for checking.

Activity 2

1 In pairs, students deal themselves four single-digit cards.

2. Students write down all the possible numbers that can be generated using those four digits.

3. Students then each order their numbers from smallest to largest and record how many groups of 100 could be formed from the smallest number. Pairs of students swap their solutions for checking.

Climb the ladder

Counting On **framework reference: Level 1**

Purpose

To help students order numbers with 2-digits.

Materials needed

Digit cards 0–9.

How to play

1. Place digit cards face down in a pile.

2. Players take turns to turn over two digit cards. The player decides which digit is the tens and which is the ones.

3. Each student draws a five square ladder as below. Students write the number they have made onto one "rung" of their ladder.

8 = tens
3 = ones ... 83

4. The ladder must progress from smallest to largest numbers, from bottom to top.

5. If you cannot put your number on the ladder in order, you must lose that turn.

6. First one to complete their ladder wins.

Extension

3-digit numbers.

Highest number

Counting On **framework reference: Level 1, Level 2**

Purpose

To help students order numbers with 3-digits or 4-digits.

Materials needed

Dice: one for each pair of students (1–6 or 0–9 dice may be used); scoring sheets: one for each pair of students

Time required

One lesson.

Instructional sequence

1. The teacher and a student (or two students) demonstrate the game on the chalkboard.

2. Students play in pairs, sharing one score sheet. Players take turns to roll a die to try to make the highest number they can. Once a number has been placed in a column its position cannot be changed. The student who makes the higher number wins that game.

3. Students play several games to determine an overall winner.

4. The teacher ties the lesson together by asking, *What is the largest possible number you can score*? (9999 if you are using 0–9 dice and playing a 4-digit game.) *Who scored closest to this*? *What was your highest number*? *What was your lowest number*?

5. Some of the results may be written on cards and pinned onto a "clothesline" to help students order 3-digit and 4-digit numbers.

Variations

1. Use 1–6 dice or 0–9 dice.
2. Total numbers after several games.

Extension

See "The nasty game".

Highest number

Score sheet

Player's name: _____		Player's name: _____	
Game		Game	
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
7		7	
8		8	
9		9	
10		10	

The nasty game

Counting On **framework reference: Level 1, Level 2**

Purpose

To help students order numbers with 3 or 4 digits.

Materials needed

Dice: one for four players (1–6 or 0–9 dice may be used); scoring sheet: one for four players

Time required

One lesson.

Rules

1. This game must be played with four players and four games must be played. One player records the rolls and the scores.

2. The rules are similar to "Highest Number" except that players are allowed to place the numbers they roll in their opponents' squares. For example, a player may place a "1" in an opponent's hundreds column. Note: Players must explain to the scorer where they want to place the number they have rolled "*Put the 2 in Susan's hundreds column*."

3. The winner of each game scores 4; 2^{nd} = 3; 3^{rd} = 2; 4^{th} = 1. Therefore, after the first game players should use various strategies to ensure that the winner of the first game does not win again. Players who really understand this game should base their strategies on the progressive scores after each round. Note: Each player must have a turn at going first.

Variation

Use 1–6 or 0–9 dice.

Extension

To encourage higher order thinking introduce the rule that players may not place the numbers they roll in their own squares.

The nasty game

Name	**Nasty game 1**	Score
_____	☐☐☐	_____
_____	☐☐☐	_____
_____	☐☐☐	_____
_____	☐☐☐	_____

Name	**Nasty game 2**	Score
_____	☐☐☐	_____
_____	☐☐☐	_____
_____	☐☐☐	_____
_____	☐☐☐	_____

Name	**Nasty game 3**	Score
_____	☐☐☐	_____
_____	☐☐☐	_____
_____	☐☐☐	_____
_____	☐☐☐	_____

Name	**Nasty game 4**	Score
_____	☐☐☐	_____
_____	☐☐☐	_____
_____	☐☐☐	_____
_____	☐☐☐	_____

Player's name	Rounds				Total points
	1	2	3	4	

0	1
2	3
4	5

6	7
8	9
10	11

12	13
14	15
16	17

18	19
20	21
22	23

24	25
26	27
28	29

30	31
32	33
34	35

36	37
38	39
40	41

42	43
44	45
46	47

48	49
50	51
52	53

54	55
56	57
58	59

60	61
62	63
64	65

66	67
68	69
70	71

72	73
74	75
76	77

78	79
80	81
82	83

84	85
86	87
88	89

90	91
92	93
94	95

96	97
98	99
100	

Notes

Activities to support levels of conceptual development in multiplication and division

Multiplication and division strategies

Level	What students can do	Students are moving to
Level 1 Forming equal groups	• Use perceptual counting and sharing to form groups of specified sizes. • Count perceived items by ones.	• Forming equal groups. • Using perceptual skip counting to find the total of equal groups.
Level 2 Perceptual multiples	• Can use multiples in perceptual counting and sharing. • Equal groups are modelled and counted using rhythmic counting and skip counting.	• Counting forwards and backwards using a pattern of multiples. • Counting equal groups without individual items being visible.
Level 3 Figurative units	• Can calculate composites when screened. • Can calculate composites only with reference to perceptual markers.	• Being able to use a composite unit a specific number of times. • Counting forwards and backwards using a pattern of multiples.
Level 4 Repeated abstract composite units	• Can coordinate composite units in repeated addition and subtraction using the unit a specified number of times.	• Coordinating two composite units as multiplication and division. • Using multiplication and division as inverse operations.
Level 5 Multiplication and division as operations	• Can coordinate two composite units. • Can solve partially screened array tasks using multiplication and division. • Can calculate division tasks through knowledge of relevant multiplication fact.	• Being able to extend knowledge of number facts to derive other relationships, e.g. 20 x 8, 20 x 80, 20 x 800 12 x 13 = 10 x 13 + 2 x 13

Teaching multiplication and division

An effective method of teaching multiplication and division is to teach for:

- **Understanding**: using concrete materials and visual, hands-on activities to give student opportunities to focus on concepts such as equal groupings or repeated addition and an understanding of just what multiplication represents.

- **Connecting**: looking for patterns in multiplication tables, related facts and reversibility.

- **Recall and practice**: recalling and using multiplication facts in games and other activities.

This set of activities for multiplication and division emphasises these three principles.

Effective teaching

- Making groups: Forming equal groups, such as groups of five, is the basis of multiplication and division.

- The structure of an array provides a powerful visual representation of multiplication with obvious links to area.

- Building on the link between multiplication and area, investigating factors from rectangles support the ideas of factors, multiples, square numbers, prime numbers and composite numbers.

- Multiplication grids can provide the stimulus for students to find patterns in multiplication tables.

- The activities designed to consolidate recall of multiplication facts or the habituation of knowledge are usually presented as games.

- Many of the activities are designed to support small group work. This encourages discussion and the use of appropriate mathematical language. (Mathematical communication is addressed in working mathematically outcomes WMS 2.3, WMS 3.3, WMS 4.3)

Unit 1: Forming equal groups

Multiple count

Stage 3

Counting On **framework reference: Level 1**

Purpose

To practise using a multiple count.

Materials needed

Classroom chairs.

Time required

20 minutes.

Instructional sequence

1. Arrange three chairs at the front of the room. Organise one student to record the sequence of multiples created on board.
2. The teacher sits in the first chair and counts out loud: "1, 2, 3...". The student records "3" on the board.
3. The teacher moves to the next chair and counts: "4, 5, 6…" . The student records "6".
4. The teacher moves to the next chair and counts: "7, 8, 9…". The student records "9".
5. The teacher asks: *If we add another chair, what would the count be*? If students are not able to predict the multiples, continue adding chairs and developing the multiple count up to 30.
6. *If we had six chairs what would the count be*?
7. Students practise counting by threes forwards and backwards. Chairs can be used as prompts for the multiples.

Variation

This activity can be used to introduce and reinforce any multiple count.

Extension

1. Calculator. Use the constant addition function on the calculator to practise counting by multiples. To count by twos you might enter the following sequence:
   2 + +, =, =, =, =,...
   Note some calculators use a different sequence to activate the constant function.

Rectangular grids

Counting On **framework reference: Level 1**

Purpose

To encourage the development of skip counting.

Rationale

Grids are a useful way of strengthening students' understanding of multiples and factors of numbers and of developing skip counting techniques.

Materials needed

1 cm grid paper.

Time required

30 minutes.

Instructional sequence

1. Draw or cut from squared paper a rectangular grid and number the squares. For example, using a 3 x 4 grid, the right-hand column shows the multiples of 4.

1	2	3	4
5	6	7	8
9	10	11	12

2. Count the squares using a rhythmic counting approach. This requires a stress on the multiple. For example, count these squares as 1, 2, 3, **4**, 5, 6, 7, **8**, 9, 10, 11, **12**.

3. Gradually introduce skip counting, i.e. 4, 8, 12.

4. Ask: "How many 'fours' in 'twelve'?" "How do you know?"

5. Repeat for other rectangular grids.

Card capers

Stage 3

Counting On **framework reference: Level 1**

Purpose

To develop the concept of a composite group.

Time required

1 lesson.

Materials needed

Multiple sets of array strips. Sets of cards as transparencies for use with an OHP.

Instructional sequence:

1. Use the cards to practise skip counting. Use the overhead projector to keep adding cards one at a time as students count in multiples. Ask the question, *How many... can you see*?

2. Use a number of cards to model an array on the OHP. Then ask students to make their own. Ask, *Make three lots of six. Which shape will you use? How many will you have altogether*?

3. Ask some students to make an array one way, such as two groups of five and other students to make an array a different way such as five groups of two. Compare the totals and discuss why they both have the same total.

4. Set challenge questions: *How could you make a total of 12 using the same card each time*? *How could you make a total of 10 using the same card each time*? Students then record the different combinations they have made.

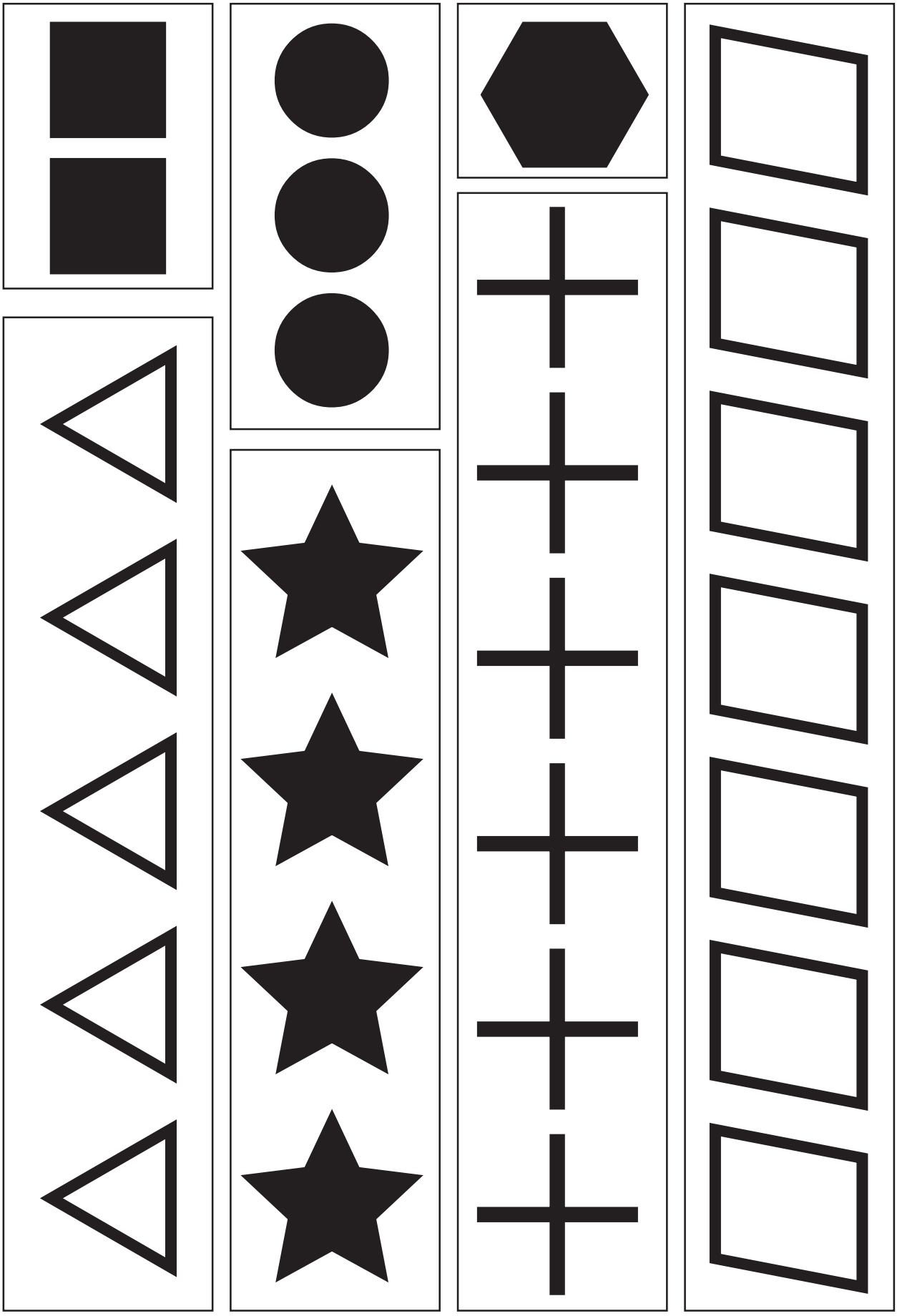
5. Multiplication game

   Materials: sets of array strips from page 134, sets of cards with multiplication facts such as 3 groups of 4, 6 lots of 2, 5 x 4.

   Instructions: Multiplication cards are placed face down in the centre of the group. The other cards are in separate piles according to size. Players take turns to turn over a multiplication card and use the other cards to construct the array. The player works out the total and records it as his or her score for that round.

   Variation: two dice can be used instead of multiplication cards.

Array strips



Blobs and rectangles

Counting On **framework reference: Level 1**

Purpose

To help students see the connection between multiplication facts and area questions.

Materials needed

1 cm grid paper for students.

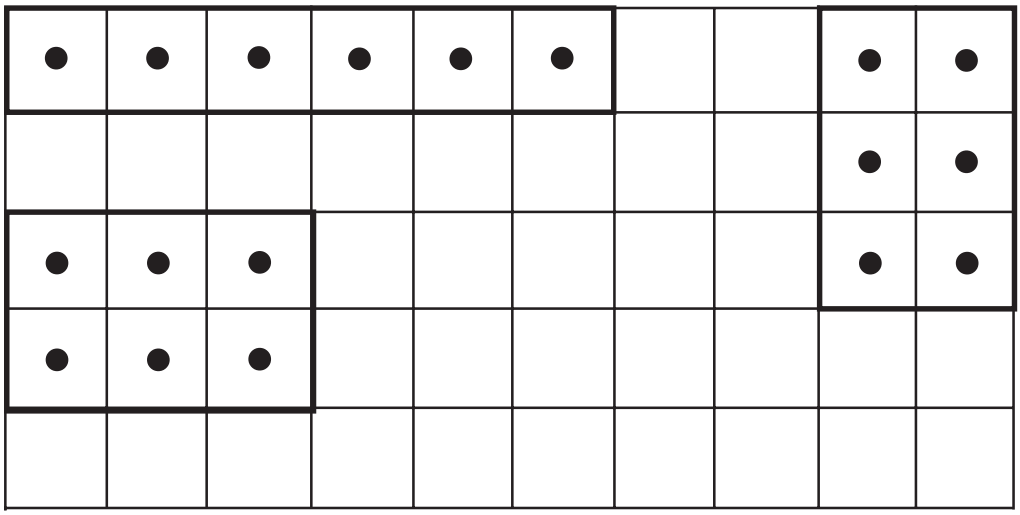
Overhead transparency of a 1 cm grid.

Time required

One or two lessons.

Instructional sequence

1. Using 1 cm grid paper, ask students to draw as many rectangles as possible which contain six dots. Ask students to draw one dot per square.
2. After a few minutes ask students to volunteer to draw one of their rectangles on the overhead transparency. Encourage students to look for as many solutions as possible. Answers might include:



Request that students mark a thick edge around their rectangles.

3. Ask your students to suggest ways of representing the number of dots, for example, "One row of six dots; two rows of three dots; three rows of two dots". Represent these with symbols, 1 x 6; 2 x 3; 3 x 2.
4. Repeat using eight dots, twelve dots, sixteen dots, and eighteen dots.
5. This activity can be extended to investigate prime numbers, say, 1 to 30. Which numbers have exactly two rectangles (such as one row of seven dots or seven rows each with one dot)?

Notes

Unit 2: Perceptual multiples

Hundreds chart

Counting On **framework reference: Level 2**

Stage 3

Purpose

To investigate number patterns with multiples on the hundreds chart.

Rationale

Frequent opportunities to practise computing, reciting and recording number sequences will assist students to become competent with using multiples.

Time required

One lesson.

Materials needed

Hundreds chart for an OHP, transparent counters, individual hundreds charts and counters for students.

Instructional sequence

1. Using the hundreds chart on the overhead projector, ask the students to state the multiples of a nominated number, such as three. Use counters to build up the pattern. ...3, 6, 9, 12, 15, 18.
2. Ask students to describe the pattern the multiples of 3 has made, e.g. diagonal pattern.
3. Ask questions about the pattern, e.g. "How many 3s in 18?" and demonstrate by counting the 6 groups of 3 to get to 18 and indicating the 6 counters on the chart.
4. Allocate different multiples from 2s to 10s to pairs of students and ask them to find the patterns. Patterns could be coloured with a highlighter pen.
5. Invite students to demonstrate patterns using counters on OHP. Discuss the patterns with the class.

Extension

The number facts symphony

Each group of students has certain multiples covered with transparent counters. The teacher is the conductor and begins the count from one. Each time a number from the student's set of multiples is called, those students make a sound, e.g. clap, tap a pencil on desk, click fingers (each multiple must have a distinct sound). Ask the question, "What were popular multiples when everyone played?"

Array grids

Stage 3

Purpose

Arrays provide a powerful visual representation of multiplication.

Counting On **framework reference: Level 2**

Time required

One lesson.

Materials needed

Array grid, counters, array grid and counters for OHP.

Instructional sequence

1. Model the construction of an array that matches a statement such as "Four students each have five pens".

2. Pose a problem for students to solve:

   There are 4 students. Each student has 3 sandwiches for lunch. How many sandwiches are there altogether? Students reconstruct the problem using counters to create an array and find the total by skip counting.

3. Pose other problems to enable students to be confident in organising arrays to find the solution.
   - The 10 members of the cricket team each hit 5 practice balls.
   - How many cakes were eaten if 6 people ate 2 each?
   - How many collector cards were there altogether if Sam and Joss had 9 each?
   - Five students each have 6 CDs. How many altogether?

4. Set the following task:

   A chocolate bar manufacturer needs advice on how to package a new chocolate bar. There are 16 pieces of chocolate. How might the manufacturer package the chocolate? Students use an array chart and counters to construct the array. Solutions are then recorded, e.g. 2 x 8, 8 x 2, 4 x 4.

5. Students work with a partner to construct and record a range of arrays for the following numbers: 24, 36, 40, 48.

Stage 3

Activity 1

Array dice

Time required

One lesson.

Materials needed

Array grids, counters, a pair of dice (one with dots and one with numerals).

Instructions

Students take turns to roll two dice and make up the matching array. The answer is recorded as their total for the round. The winner has the largest total after five rounds.

Variation

Students can use blank paper to draw arrays instead of using the grid.

Array grid

Array fish

Counting On **framework reference: Level 2**

Stage 3

Purpose

To develop students' understanding of multiplication and division strategies.

Time required

One lesson.

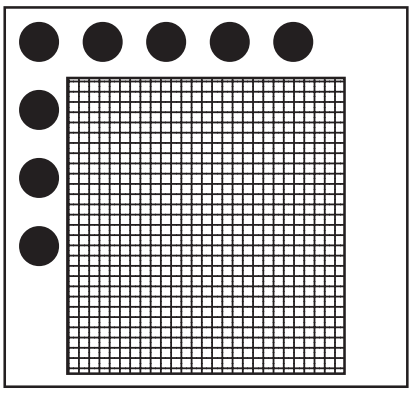
Materials needed

Array and numeral cards.

Instructional sequence

1. Whole class

   Display a partially screened array as shown below.



How many dots are there altogether?

How did you get your answer?

Give other screened arrays.

2. Small group

   Array fish

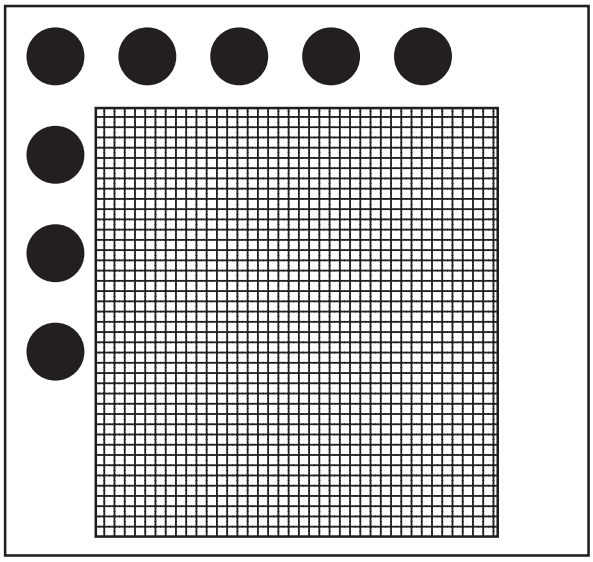
   - This is played the same as traditional *Fish*. Screened array cards are matched with numeral cards.
   - Array and numeral cards such as those shown for multiples of five, are used.
   - The teacher may choose to have students focus on particular multiples (e.g. fives) or choose cards from a range of multiples (e.g. 3, 4 and 5 etc.)

3. Whole class

   Review the lesson by showing some more screened arrays.

   Follow up with some verbal tasks:

   - *If I have 3 rows each with 5 dots, how many are altogether*?
   - *Can you describe an array with 20 dots*?



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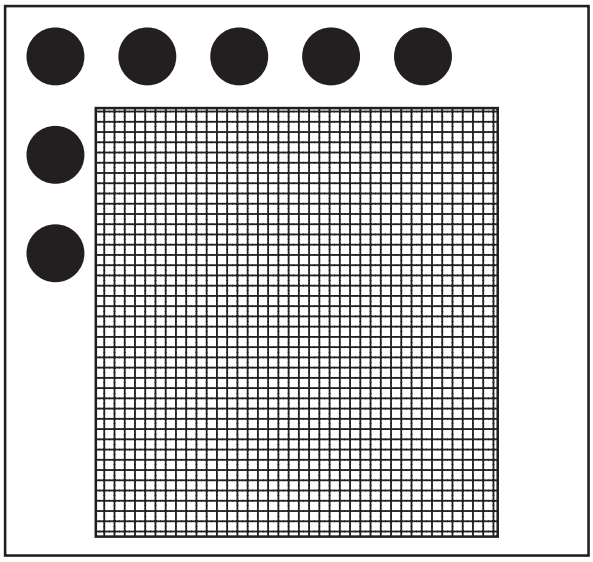

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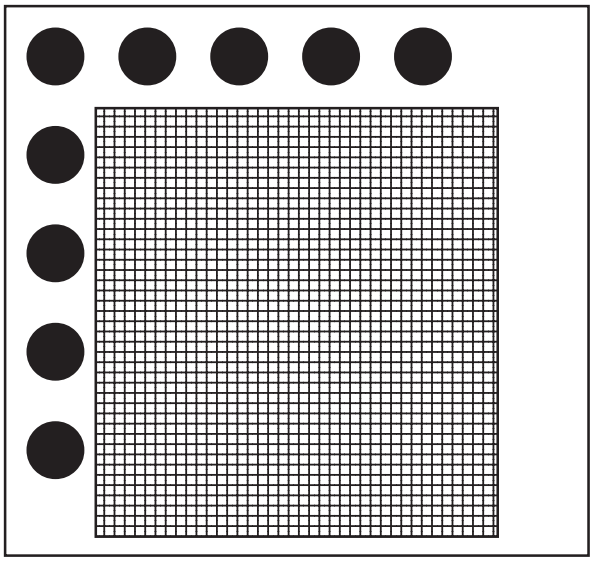


25

Arrays

Counting On **framework reference: Level 2**

Purpose

To help students to "see" what multiplication facts represent.

Materials needed

Arrays of 10 x 10 dots, one per student; pieces of cardboard to conceal dots, 2 per student. A variation is to use the 10 x 10 CD array.

Time required

One to two lessons.

Instructional sequence

1. The teacher explains the term "array" and gives students a 10 x 10 array of dots and two pieces of cardboard to cover the dots.

2. The teacher demonstrates (on the overhead projector or an 10 x 10 array taped to the chalkboard) how to form an array by using the two pieces of cardboard to cover dots and leave visible (say) a 7 x 4 array. Students can verify the answer by skip counting or knowledge of the relevant multiplication fact.

3. Students are then asked to form arrays. The teacher may ask for an array of 24 dots. Ask different students to describe how they made their arrays and record their answers as multiplication expressions. Students may discover 6 x 4, 4 x 6, 3 x 8, 8 x 3.

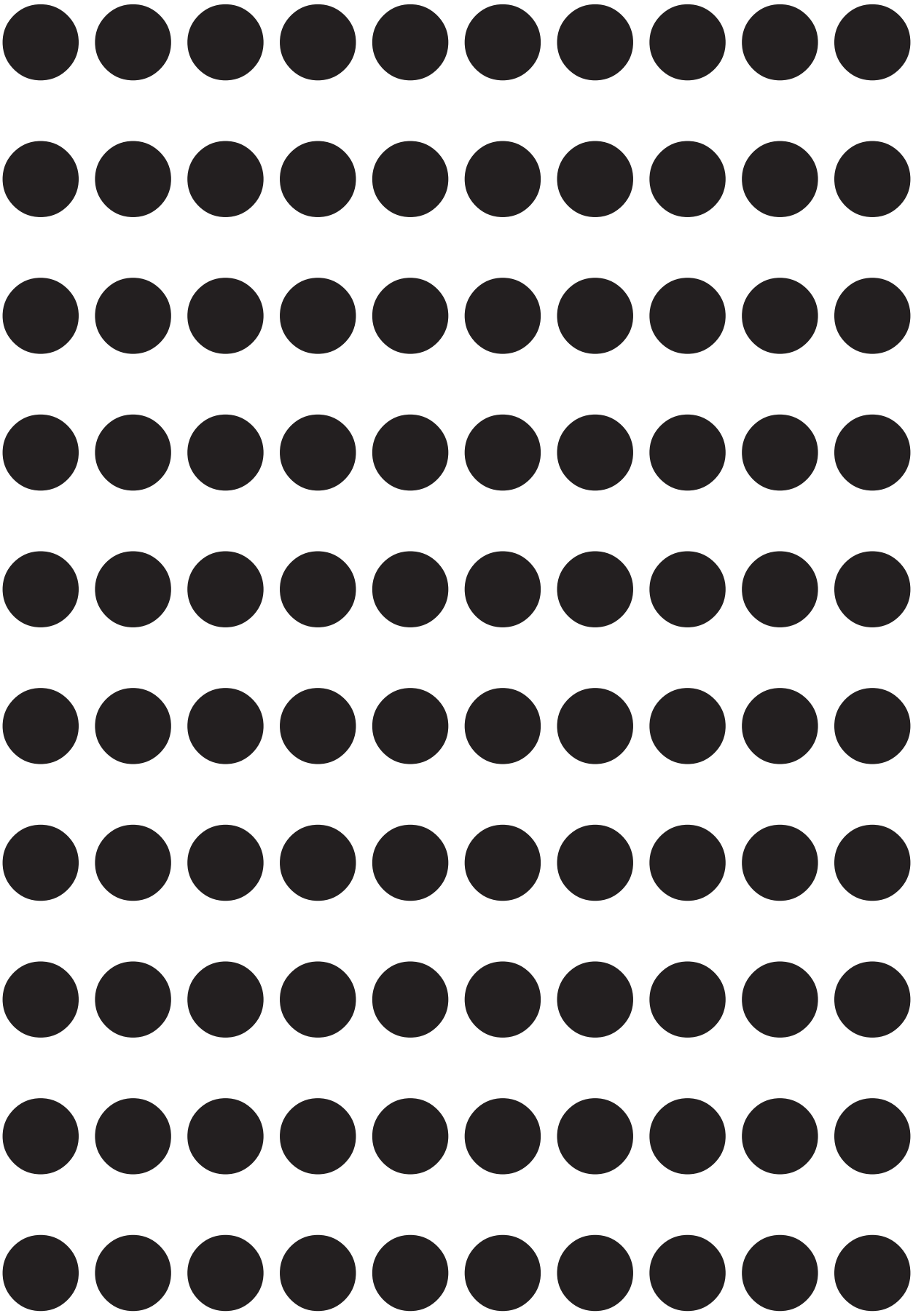
4. Students are asked to form an array of their choice and "translate" it into symbols.

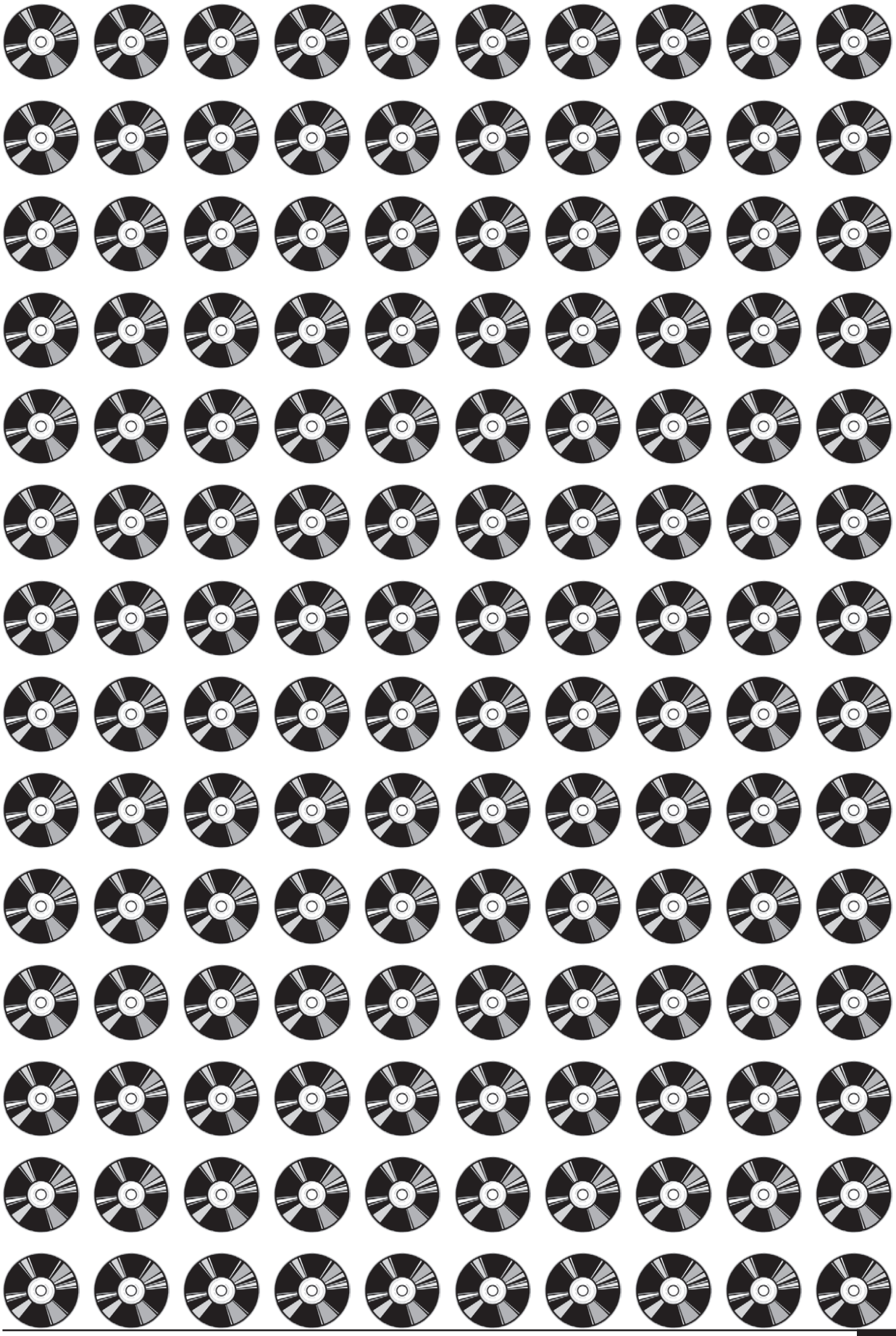
5. The teacher asks students to form an array such as 6 x 4. This array is then discussed in terms of both multiplication and division: 6 x 4 = 24; 4 x 6 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6. This helps students to see the link between multiplication and division. Students may also create word problems to match the array. For example:
   (i) 6 books @ $4.00 each = $24.00
   (ii) If I have 24 lollies, can I give 6 students 4 each?

6. Students may use the array to form square numbers and find these in multiplication tables or on a multiplication grid.

7. Students may form arrays and turn them through 90^0 to demonstrate that multiplication is commutative.

Array dots





Division array

Purpose

To develop the link between multiplication and division. To move towards divisions with remainders. To carry out division with tens.

Counting On **framework reference: Level 2**

Materials needed

10 x 10 division array for each student and one for the OHP. Two pieces of cardboard.

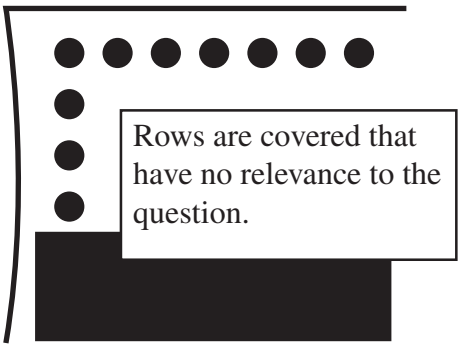
Time required

One lesson.

Instructional sequence

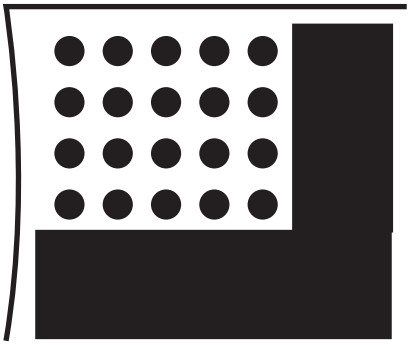
1. Use the OHP to model a division array for 20 ÷ 4. Ask the question, I have 20 KitKats to share with 4 students. How many will each get?

2. Cover the bottom of the array so that only four rows are visible.



3. Move the second piece of cardboard from the right and ask the question, "Four lots of what makes twenty?" The number of columns specifies the solution.

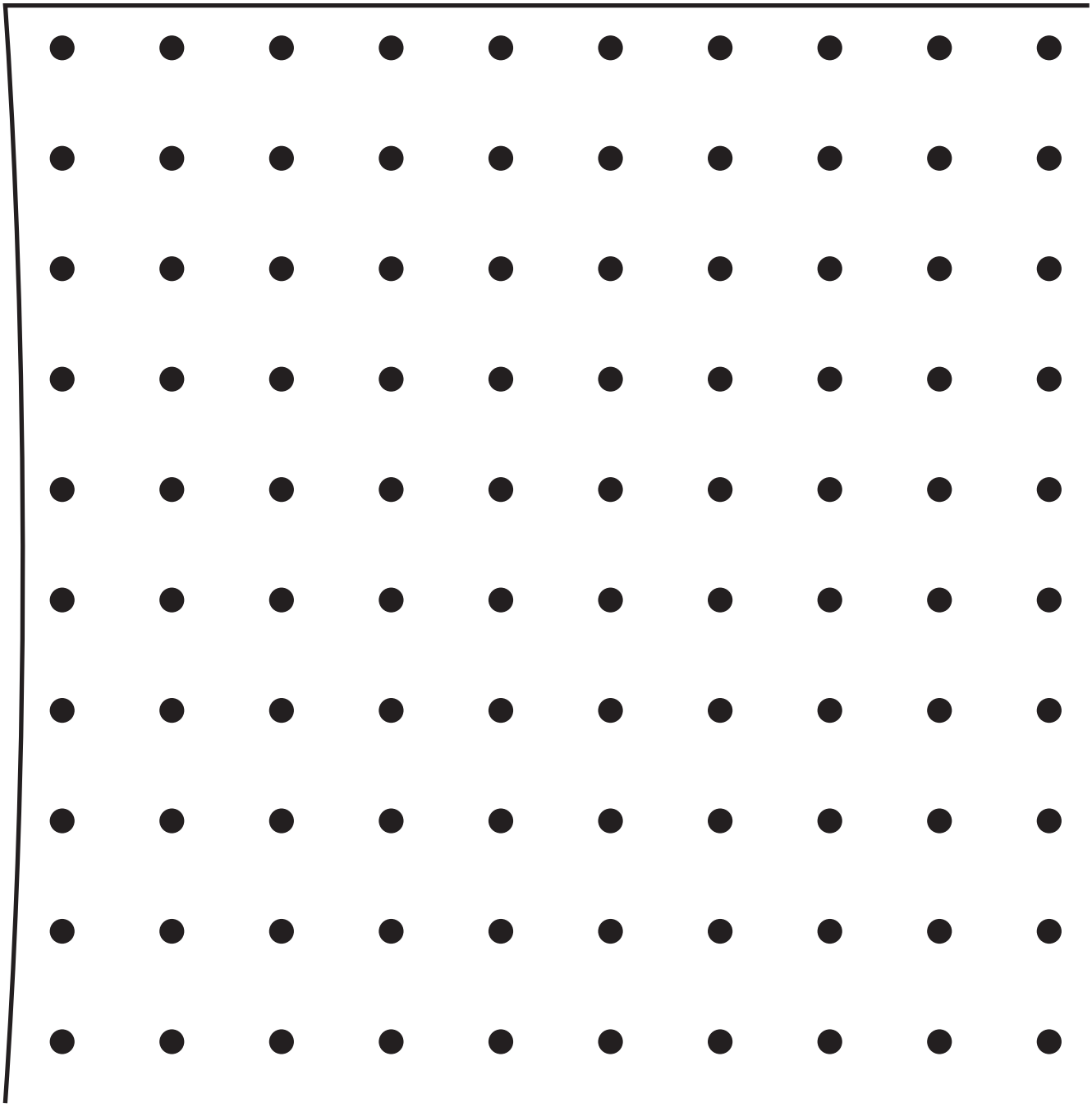
20 ÷ 4 = 5



Student determines the total number of dots (20) in the array by multiplicative processes.
The number of columns is the solution to the division.

4. Further examples are given and students complete arrays using their own array grid.

10 by 10 division array



Partially covered arrays

Counting On **framework reference: Level 2**

Purpose

To provide practice in reconstructing arrays where all of the individual units are not visible.

Time required

One lesson.

Materials needed

Worksheets

Instructional sequence

1. Whole class

   Introduce the black and white mat task as a poster problem. Provide time for students to discuss the strategies they used to obtain the answer.

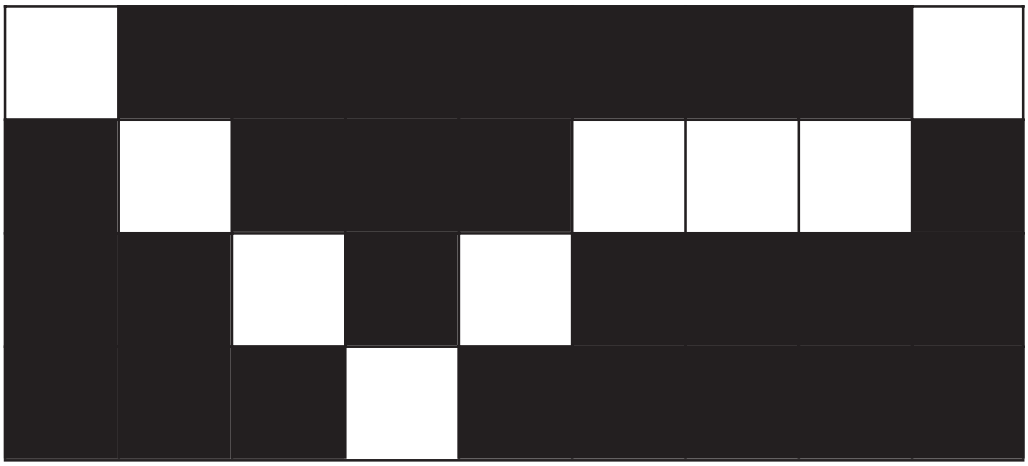
2. Partner activity

   Students work with a partner to discuss and record solutions for all of the problems.

3. Students may like to design other partially covered array problems for other students to solve.

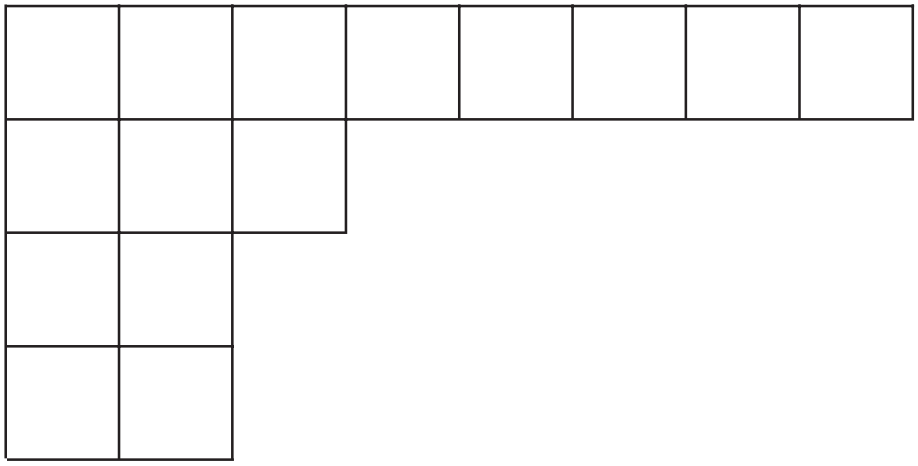
The black & white mat

I made a patchwork mat with some black squares and some white squares all the same size. How many squares did I use altogether?



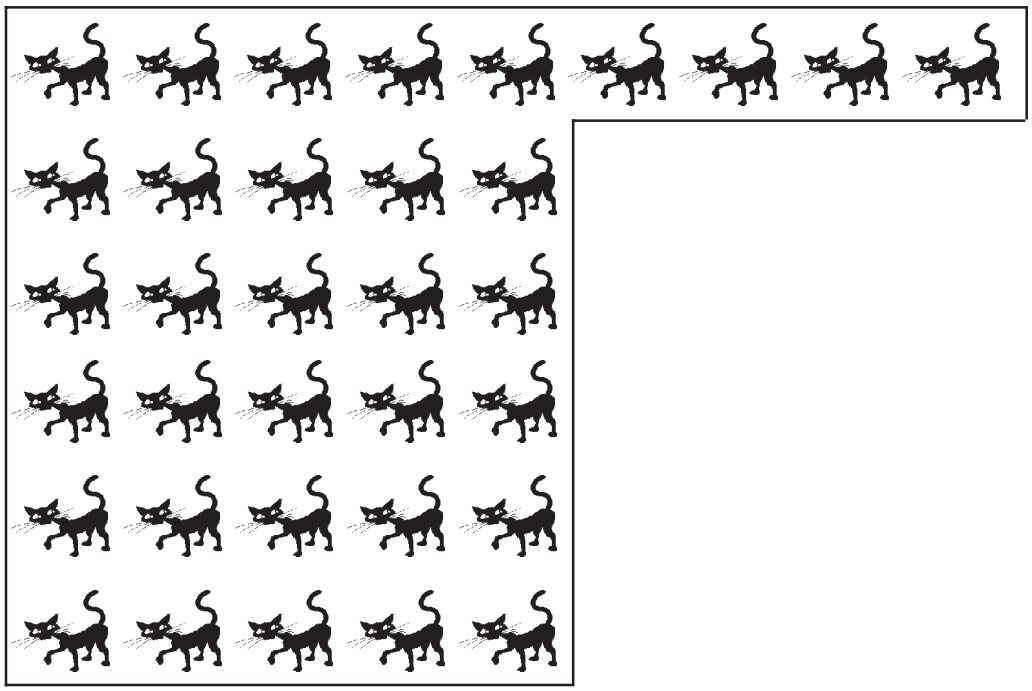
The block of chocolate

My brother ate some of my block of chocolate. How many squares were in my block of chocolate before he ate some?



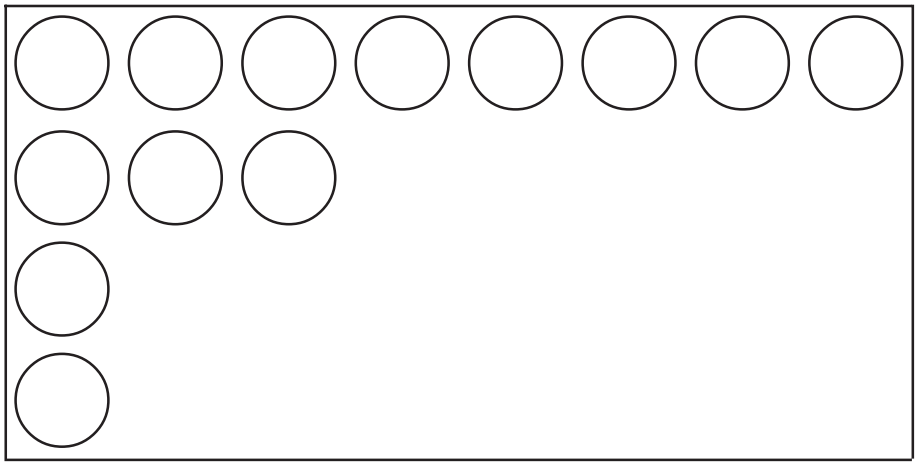
The wrapping paper

I bought some wrapping paper with cats on it. I cut a piece of the paper to wrap my mother's present. How many cats were on the whole sheet of paper before I cut it?



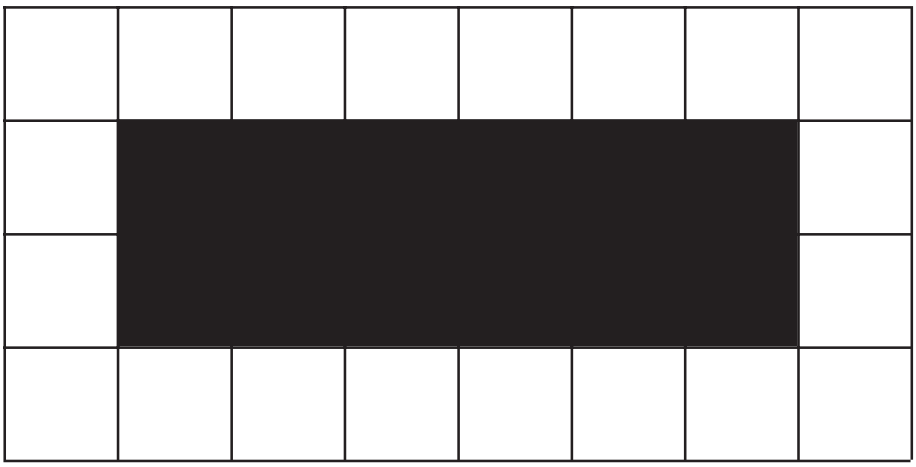
Biscuits

This tray had several rows of biscuits but some biscuits have been taken.
How many biscuits were there when the tray was full?



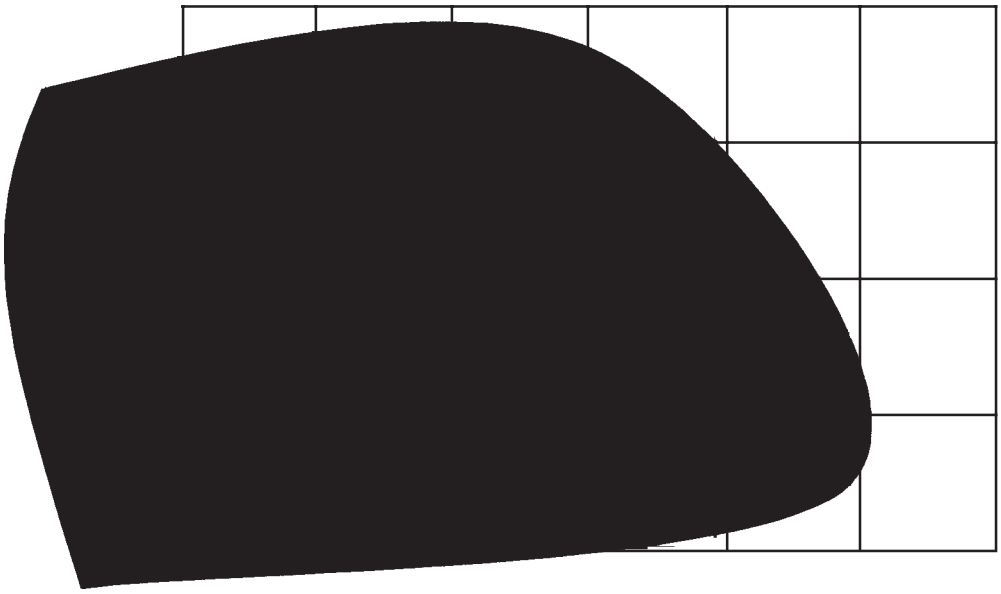
Tiles

This is the bathroom floor with a black bathmat on it.
How many tiles are on the whole floor?
How many tiles are covered by the bathmat?



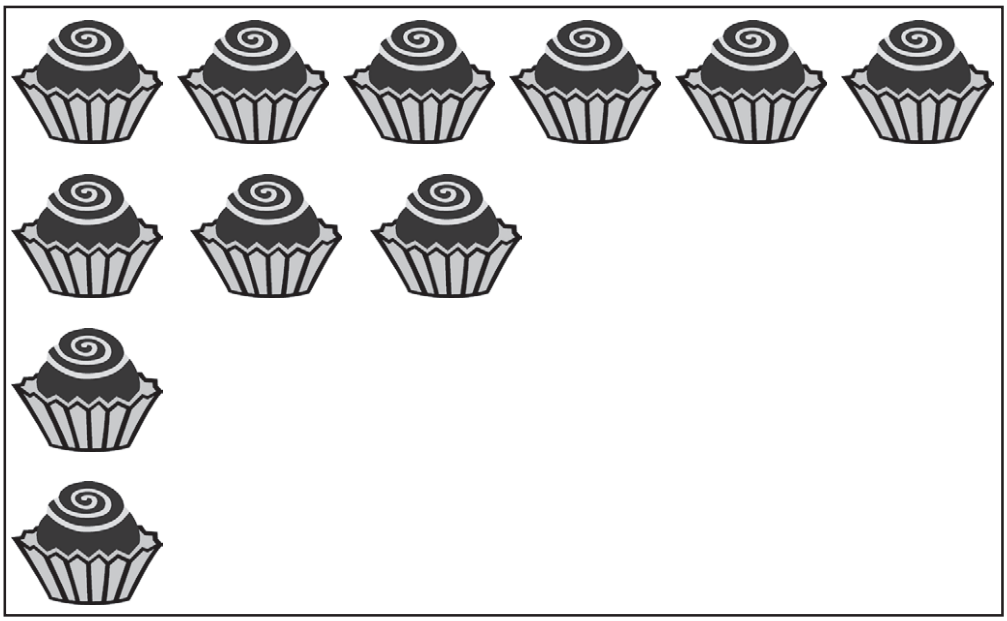
The patchwork quilt

I have a patchwork quilt. Here it is with a blanket covering part of it.
How many square patches are there in my quilt?



The box of cakes

Dad bought a box of cakes for my party. The cakes were arranged in rows in the box. Here is the box of cakes after we all ate some. How many cakes did Dad buy?



Notes

Unit 3: Figurative units

Multiplication grids

Counting On **framework reference: Level 3**

Purpose

To consolidate number facts by investigating patterns in multiplication tables.

Materials needed

10 x 10 multiplication grids, one per student and one overhead transparency.

Time required

One lesson.

Instructional sequence

1. Distribute the 10 x 10 multiplication grid.

2. Ask the following questions:

   The number "20" appears on the grid four times. Why? What other numbers appear four times? Are there any numbers which appear only once? Are there any numbers less than 100 which don't appear at all? Why?

   Ask your students to imagine covering a 10 x 10 array with two pieces of cardboard, just as they did in the activity "Arrays". Picture forming a 2 x 2 square and then a 3 x 3 square and so on. Now ask your student to find the "squares" on the multiplication grid. Students can find the squares on the left-right diagonal of the grid.

3. Show the overhead transparency of the multiplication grid and invite students to shade in the multiplication facts they know, starting from the results from multiplying by one.

   If a student knows 1x, 2x, 5x, 10x, only 36 facts remain. Of these, 6 are squares. There are only 15 facts remaining because of "reversals". Students can find these reversals on diagonals within the grid.

   If a student has learned 1x, 2x, 10x, 5x, 4x, 3x, there are only 16 facts remaining, including 4 squares and 6 reversals.

   Introduce questions on division as each multiplication table is identified. For example, "Which multiplication lets me answer 24 ÷ 4?"

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

x	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

Factors from rectangles

Counting On **framework reference: Level 3**

Purpose

To introduce the idea of factors and to consolidate the link between multiplication and area.

Materials needed

1 cm grid for each student or cardboard squares.

Time required

One lesson.

Instructional sequence

1. Students are issued with grid paper to draw rectangles or cardboard squares to form rectangles.

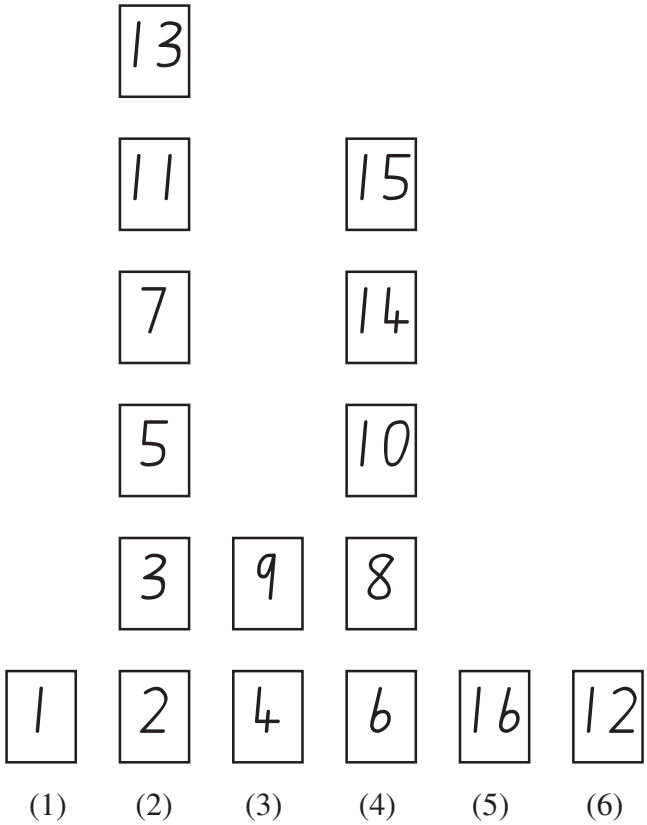
2. The teacher assigns a number from 1 to 16 to each pair of students. You might need to assign two numbers to some students. For example, 1 and 12 might be a good pair. Students are asked to draw or make rectangles that contain their assigned number of squares. Ask the class to decide if you could make a 3 x 3 rectangle for the number nine. Gain consensus that a square is just a special rectangle.

3. Students record the lengths of the sides of the rectangles they have drawn or made. In the circle on the worksheet they record the number of different lengths they used. Students who are assigned the number "6" will use the lengths "1", "2", "3", "6" and will therefore write the number "4" (lengths) in the circle.

4. Students then form groups with others who have written the same number in the circle.

5. The teacher then guides the students to make a "graph" on the floor with the sheets.
   - Only one pair of students will have written the number "1".
   - Six pairs of students will have written "2"; the students who were assigned 2, 3, 5, 7, 11, 13
   - Two pairs of students will write "3"; (4, 9)
   - Five pairs of students will write "4"; (6, 8, 10, 14, 15)
   - One pair of students will write "5"; (16)
   - One pair of students will write "6"; (12)

The graph will look like this:



6. Facilitate a whole–class discussion on the reasons for the different numbers (factors, multiples, squares, primes, etc.) This activity can be used to show that "1" is neither prime nor composite. Only one pair of students will have written "1" in the circle. This demonstrates that "1" is the only number which has only one factor.

   The second column of the graph contains only one even number, two. Two is the only even prime number. Why is this so?

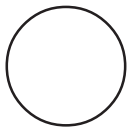
   Students should be able to work out from the 1st, 3rd and 5th columns that square numbers have an odd number of factors. Why is this so?

Extension

Students will probably wish to extend this investigation beyond 16.

Students could investigate the question: "Which number less than 100 has the most factors?"

Draw as many rectangles and squares as you can that contain _____ squares.
Write the lengths of the sides on the shapes. Write the numbers you used to write the lengths here: _____

How many **different** numbers did you use? 

Find other people who had the same number written in the circle.

Notes

Unit 4: Repeated abstract composites

Dice times

Counting On **framework reference: Level 4**

Purpose

To consolidate multiplication facts up to 6 x 6 in a simple investigation.

Materials needed

Two dice (1–6) for the teacher.

Time required

One lesson.

Instructional sequence

1. The teacher rolls two dice and the numbers are multiplied together (say) 5 x 4 = 20.

2. The teacher rolls the dice again and the students work out the product.

3. The teacher asks students to suggest other possible results on rolling two dice and multiplying the numbers together.

4. The teacher poses the question, *If I keep rolling two dice, how many different numbers could I make by multiplying the numbers together*?

5. Students work in pairs to work out all the possible combinations.

6. The teacher collates the responses until the 18 possible solutions are found.

7. The teacher facilitates class discussion of methods used. If one of the dice shows "1", the possibilities are 1, 2, 3, 4, 5, 6. If one of the dice shows "2", the possibilities are 2, 4, 6, 8, 10, 12. (Students should note that "2", "4" and "6" have already appeared as 1 x 2, 1 x 4 and 1 x 6.).

   Ask students to display or explain the strategies they used to ensure they discovered all of the possible products, e.g. make a table, start at the smallest, eliminating numbers which have already appeared and checking tables grids.

Dice tables

Counting On **framework reference: Level 4**

Purpose

To consolidate multiplication facts up to 6 x 6. This activity can follow the investigation "Dice times".

Materials needed

One dice tables grid for each pair of students; three dice (1–6) for each pair of students; counters of two colours (12 of each) for each pair of students.

Time required

One lesson.

Rules

1. Students play in pairs. Each student has twelve transparent counters of one colour.

2. Students take turns to roll the three dice. The player chooses two of the numbers and multiplies them together to make a number on the board and covers it with a counter, e.g. a student who rolls 2, 3, 5 could make 2 x 3 = 6 or 2 x 5 = 10 or 3 x 5 = 15.

3. The winner is the first student to get 4 in a row, column, diagonal or square.

Questioning

1. Teachers may ask, "If I roll 3, 4, 5 what numbers can I make?"

2. Teachers may ask, "If I want to claim a '12' on the board, how could I get it? Is there another way?"

Dice tables

4	16	5	36	6
24	1	30	10	12
6	12	20	24	18
15	8	25	2	6
3	18	4	12	9

Roll three dice. Choose two numbers and multiply them.

Dice tables

4	16	5	36	6
24	1	30	10	12
6	12	20	24	18
15	8	25	2	6
3	18	4	12	9

Roll three dice. Choose two numbers and multiply them.

Four in a row or four in a square

Counting On **framework reference: Level 4**

Purpose

To consolidate students' familiarity with the 2x, 3x, 4x, ... 9x tables.

Materials needed

A set of cards from 1 to 10.

Sets of cards for each multiplication table

Counters of four colours (10 of each), one colour for each student

Time required

One or two lessons.

Instructional sequence

Activity 1

1. In groups of 3 or 4, one student turns over a card from the 1–10 set.
2. The student multiplies the number by 2 (or 3 if using the 3 times table, etc.). For example, if a student turns over the number 4, the student solves 2 x 4.
3. The student places a counter on the answer in the corresponding times–table grid.
4. The next student has a turn.
5. The winner is the first player to get four adjacent squares in a row, column, diagonal or square.

Activity 2

All tables

For this activity you will need two or three sets of cards from 1 to 10.

1. Groups of 3 or 4 students are dealt 5 cards each.
2. The first student multiplies two of the cards together, say 9 x 5, and then covers a square with a counter. The student then picks up another two cards so that there are always five cards with each student.
3. The next student has a turn. The winner is the first student to get 4 in a row, column, diagonal or square.

Extension

Use blank pro-forma on page 176 for students to construct their own game.

2 x Table

6	18	10	16	2	20
20	12	4	14	18	10
14	16	10	2	6	12
8	2	20	16	4	14
4	6	8	4	12	10
18	10	18	8	16	2
12	6	2	20	8	14

3 x Table

9	27	15	24	3	30
30	18	6	21	27	15
21	24	15	3	9	18
12	3	30	24	6	21
6	9	12	6	18	15
27	15	27	12	24	3
18	9	3	30	12	21

4 x Table

12	36	20	32	4	40
40	24	8	28	36	20
28	32	20	4	12	24
16	4	40	32	8	28
8	12	16	8	24	20
36	20	36	16	32	4
24	12	4	40	16	28

5 x Table

15	45	25	40	5	50
50	30	10	35	45	25
35	40	25	5	15	30
20	5	50	40	10	35
10	15	20	10	30	25
45	25	45	20	40	5
30	15	5	50	20	35

6 x Table

18	54	30	48	6	60
60	36	12	42	54	30
42	48	30	6	18	36
24	6	60	48	12	42
12	18	24	12	36	30
54	30	54	24	48	6
36	18	6	60	24	42

7 x Table

21	63	35	56	7	70
70	42	14	49	63	35
49	56	35	7	21	42
28	7	70	56	14	49
14	21	28	14	42	35
63	35	63	28	56	7
42	21	7	70	28	49

8 x Table

24	72	40	64	8	80
80	48	16	56	24	40
56	64	40	8	32	48
32	8	80	64	16	56
16	24	32	16	48	40
72	40	72	56	64	8
48	24	8	80	32	56

9 x Table

27	81	45	72	9	90
90	54	18	63	81	45
63	72	45	9	27	54
36	9	90	72	18	63
18	27	36	18	54	45
81	45	81	36	72	9
54	27	9	90	36	63

All tables playing card multiplication board

You need

3–4 players
Cards 1–10
Counters of four colours (10 of each)

Rules

Deal five cards to each player. Multiply 2 cards together, e.g. 9 x 5 = 45
Cover a square that has that number with one of your counters. Replace the two cards you used. (Always have five cards.)
Next player has a turn.
The winner is the first player to get 4 in a row, column, diagonal or square.

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
50	54	56	60	63	64
70	72	80	81	90	100

☐ x tables

You need

3–4 players
Cards 1-10
Counters of four colours
(10 of each)

Rules

One player turns up a card.

Multiply that number by ☐

Cover a square that has that number with one of your counters
Next player has a turn.
The winner is the first player to get 4 in a row, column, diagonal or square.

Unit 5: Multiplication and division as operations

Multo

Counting On **framework reference: Level 4, Level 5**

Purpose

To consolidate multiplication facts and to investigate factors in determining a winning strategy.

Materials needed

Flash cards containing multiplication facts from 1 x 1 to 10 x 10, Multo grids.

Instructional sequence

1. Prepare 100 flash cards with the multiplication facts 1 x 1 through to 10 x10.

2. Students are given a 4 x 4 grid in which they must write 16 different numbers. The winner is the first student to get four numbers in a row, column or diagonal. On completion of a row of 4 the winning student calls out "Multo!"

3. As each flash card is shown, students cross off that product from their game boards. The teacher may decide to have the students read the card aloud and say the answer before they check it off. This is a good way to reinforce prior learning.

4. At the completion of a game the teacher runs through the flash cards already shown and students again say the question and provide the answer. This is a check that the winner does indeed have a "correct" grid.

5. After the game has been played several times students soon discover that this activity differs from "bingo-style" games in that players can increase their chances of winning in several ways. Students work out for themselves, or with a little help from group discussion, that some numbers are "better" than others. Twenty-four is a "good" number because there are four cards which give that product (6 x 4, 4 x 6, 8 x 3, 3 x 8) whereas only one card (5 x 5) will give the answer 25.

6. If students investigate this further they may discover that there are 9 "best" numbers having four chances of being drawn (6, 8, 10, 12, 18, 20, 24, 30, 40). Four numbers have three chances: 4, 9, 16, 36. Students may decide to use the results of this investigation when choosing the numbers to place in the grid. Some students place the "best" numbers on the eight squares which occupy diagonals because they say that these squares have three chances of winning, while the other 8 squares have only two chances.

Multo

Games and activities for multiplication and division

1. Multiple relay

Materials needed

Cards printed with selected multiples.

Instructions

Place students in teams and spread the cards on the floor in front of each team. At a given signal the teams take the cards, distribute one to each member and then order themselves as a sequence of multiples. First team to finish is the winner. Some teachers like to add the rule that teams must compete in silence.

Variation

Divide the class into teams of four. Each team receives an envelope and arranges the members of the group in order of multiples, e.g. 2, 4, 6, 8 or 3, 6, 9, 12 or 5, 10, 15, 20. The team sits down when in order.

Extension

Have teams of five, give three cards per team, e.g. 4, 8, 12. The last two students continue the pattern orally by saying "16, 20".

2. Dominoes

Materials needed

1 set of dominoes, number cards 1—36.

Instructions

Spread the number cards out. Take it in turns to turn over a domino and multiply the two sets of dots. Put the domino on the appropriate number card. Continue until all dominoes are used.

Discussion: Why do some of the cards have more than one domino?

3. A trio of multiples

Materials needed

A double pack of number cards 2–30 for each group of 3, 4 or 5 players.

Instructions

Each player is dealt five cards with the remainder of the pack placed face down in the middle with the top card turned up. Players may put down any three cards which are multiples of the same number. In turn, a player will either pick up the face-up card or the top card which is face down, try to make a trio of multiples and then add one card to the discard pile. The first player to be rid of all their cards is the winner and scores 0. Other team members total the cards left in their hand. After five rounds the person with the lowest score wins.

4. Dice to 100

Students take turns to roll two dice and multiply the numbers. A cumulative total is kept by adding and the first player to reach a score of 100 is the winner.

5. Array bingo

Materials needed

Students draw a blank 4 x 4 grid and write in any 16 numbers from 1—100. The teacher has two sets of number cards 1—10.

Instructions

The teacher holds up a number from each pack and the multiplication fact is recorded on the board. Students calculate the answer and cross it out if they have the number on their grid. The first player to get 4 in a row, column, diagonal or square is the winner. Answers can be checked from list of facts on the board.

Discussion: What are good numbers to have on your board and why?

6. Calculator activities

Use the constant addition function to count by multiples, e.g. 4 [+] [+] [=] [=] [=]

Key in a starting number such as "three". Have your students press the "plus" key twice. Ask them to place a finger above the "equals" key, close their eyes and press the "equals" key as many times as they need to obtain 24 on the display. Students can open their eyes when they have finished to check if they are correct.

Have your students key in a starting number such as five and press the "plus" key twice. Ask them to predict how many times you need to press the "equals" key to make the calculator show 50 on the display.

Repeated subtraction can be dealt with in a similar way. Key in a number such as 50. Have students predict how many moves by 5 will get you to 0.

7. 3 x 3 squares

Students create a 3 x 3 square by matching multiplication questions and their answers. Instructions are included on the photocopy masters.

8. Bingo

Students play Bingo by multiplying each number drawn by two, three, four or five. Instructions are included on the photocopy masters.

9. Jigsaws

Materials needed

Copies of the jigsaw grids.

Instructions

Cut the strips either vertically or horizontally. Students use the strips to reconstruct the jigsaw.

3 x 3 square of 10 times and 5 times tables

Preparation
Copy onto light card. Cover with contact or laminate if possible. Cut into nine cards.

How to play
Students play in groups of two or three, or play individually. Place the nine cards into a 3 x 3 square, so that all sides align correctly.

Outcome NS2.3, LFN M&D 4

10 x 5 6 x 10	50 2 x 10 80	20 8 x 5
09 4 x 5 3 x 10	01 x 8 20 10 x 10 7 x 5	04 100 5 x 10
30 9 x 5	35 45 9 x 10	05 90

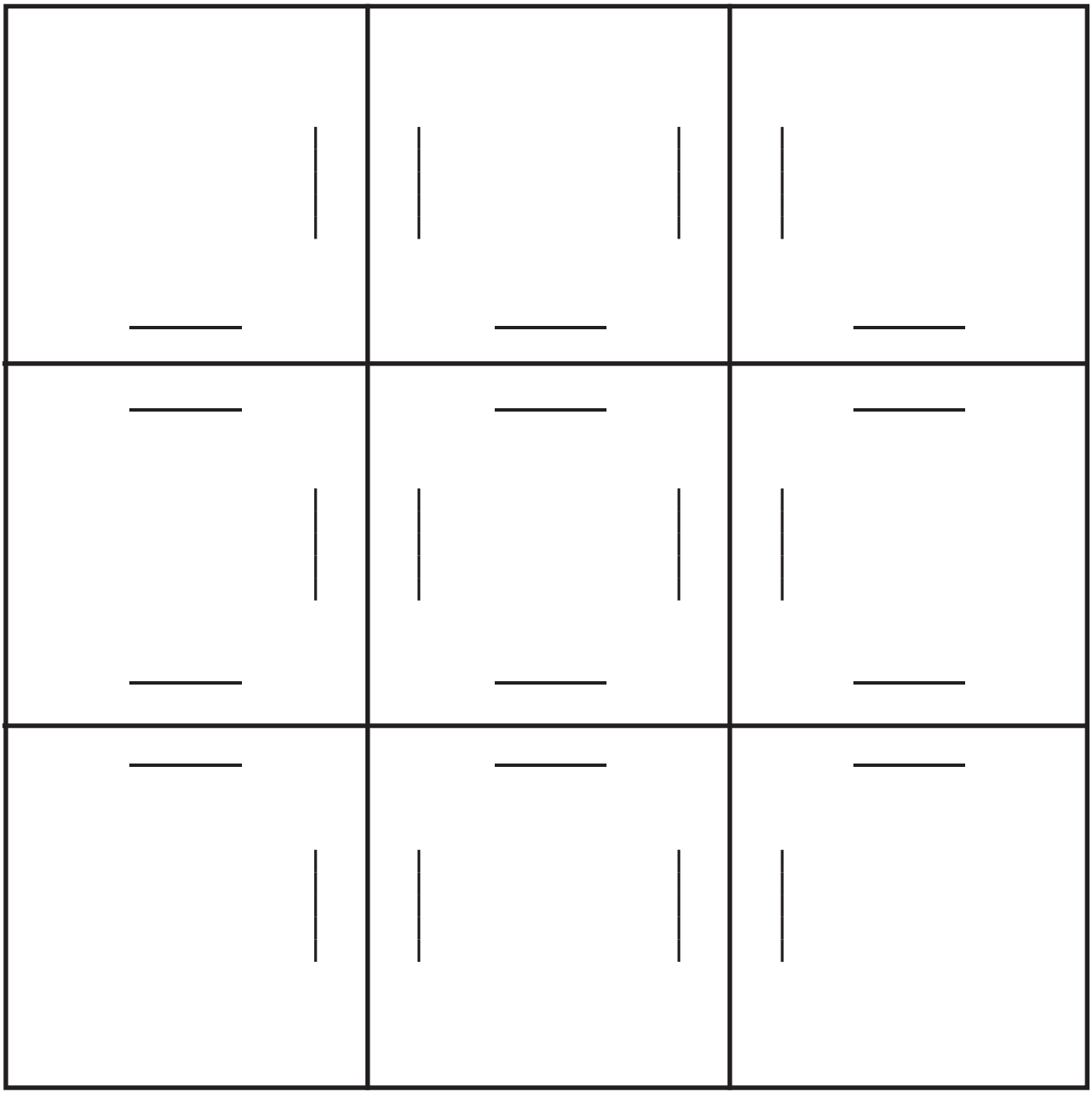
Write your own 3 x 3 square of tables

Preparation
Prepare one sheet for each student.

How to play

- Students write their own puzzles, using multiplication facts, so that an answer is opposite each question.
- Check that facts are correct by swapping with a partner or using a calculator.
- Cut on the grid lines, label the back of each piece with the author's initials, and swap with a partner to put back together.

Outcome NS2.3, LFN M&D 4



How to play

One card for each student and a ten-sided die for two or three students to share. Take turns to roll the die and multiply the number by 3. Place a counter on the answer. First to cover four numbers in a row (horizontal, vertical or diagonal) is the winner.

Outcome NS2.3, LFN M&D 4

x 3 Bingo

3	18	0	9	15
18	6	21	3	12
3	21	9	24	6
12	6	27	12	27
24	15	9	0	15

x 3 Bingo

3	18	0	9	15
18	6	21	3	12
3	21	9	24	6
12	6	27	12	27
24	15	9	0	15

How to play

One card for each student and a ten-sided die for two or three students to share. Take turns to roll the die and double the number. Place a counter on the answer. First to cover four numbers in a row (horizontal, vertical or diagonal) is the winner.

Outcome NS2.3, LFN M&D 4

Doubles bingo

16	4	18	10	12
6	2	14	0	4
12	0	4	8	18
14	2	10	6	16
0	8	16	8	10

Doubles bingo

16	4	18	10	12
6	2	14	0	4
12	0	4	8	18
14	2	10	6	16
0	8	16	8	10

How to play

One paper for each student and a ten-sided die for two or three students to share.

Students choose which multiples (e.g. three, fours or fives) to write onto their bingo paper, and fill all spaces. Take turns to roll the die and multiply the number by the chosen multiple. Place a counter on the answer. First to cover four numbers in a row (horizontal, vertical or diagonal) is the winner.

Outcomes NS2.3, LFN M&D 4

Write your own bingo

Write your own bingo

Grid for jigsaw to 200 by twos

2	4	6	8	10	12	14	16	18	20
22	24	26	28	30	32	34	36	38	40
42	44	46	48	50	52	54	56	58	60
62	64	66	68	70	72	74	76	78	80
82	84	86	88	90	92	94	96	98	100
102	104	106	108	110	112	114	116	118	120
122	124	126	128	130	132	134	136	138	140
142	144	146	148	150	152	154	156	158	160
162	164	166	168	170	172	174	176	178	180
182	184	186	188	190	192	194	196	198	200

Grid for jigsaw to 500 by fives

5	10	15	20	25	30	35	40	45	50
55	60	65	70	75	80	85	90	95	100
105	110	115	120	125	130	135	140	145	150
155	160	165	170	175	180	185	190	195	200
205	210	215	220	225	230	235	240	245	250
255	260	265	270	275	280	285	290	295	300
305	310	315	320	325	330	335	340	345	350
355	360	365	370	375	380	385	390	395	400
405	410	415	420	425	430	435	440	445	450
455	460	465	470	475	480	485	490	495	500

Notes

Activities to support Counting On students in a mainstream Classroom

Why counting on for all

Many teachers in the *Counting On* pilot expressed the opinion that all students should be exposed to the *number sense* activities.

Students who may not have been identified as needing *Counting On* who would still benefit from the activities. Students may be successfully placed in the classroom because they have been able to memorise the algorithms but have little understanding of the place value system. A strengthening of the understanding of place value for whole numbers should assist in the understanding and manipulation of decimals.

The syllabus links to the activities in this chapter extend beyond those outlined on pages 10 and 11 to include:

NS3.4 Compares, orders and calculates with decimals, simple fractions and simple percentages

The student can, for example

- read, write and interpret numbers in decimal form
- compare and order decimals (using inequality notation)
- use pen and paper methods for adding and subtracting decimals.

The activities can also be easily adapted to include links to:

NS4.3 Operates with fractions, decimals, percentages, ratios and rates

The student can, for example

- draw a number line and position fractions, equivalent fractions, decimals and percentages
- order fractions, decimals and percentages on a number line
- express percentages as a decimal.

NS4.2 Compares, orders and calculates with integers

The student can, for example

- recognise directed number as having both direction and magnitude
- order directed numbers on a number line (using inequality notation)
- add and subtract directed numbers

Unit 1: Place value considerations; moving to decimals

Decimal grids (adapted from page 22)

Counting On **framework reference: Level 4**

Purpose

To facilitate mental addition of decimals.

Materials needed

One set of the addition grids per student. These may be as per page 23 or enlarged and used as a single page per grid and be laminated to enable writing on with wipeable pens.

Instructional sequence

Add two or more numbers horizontally or vertically to make the total indicated in the circle. Some numbers are used more than once. At the end all squares should be matched.

Variation

Alter the grids to be for directed numbers with the operation of addition or subtraction.

Extension

Use the blank grid provided. Have students make their own grid and either addition or subtraction to obtain their chosen circled number.

Use the activity for operations with fractions.

1

0.4	0.6	0.1	0.6	0.3	0.8
0.3	0.7	0.6	0.5	0.2	0.2
0.5	0.4	0.2	0.5	0.4	0.1
0.2	0.6	0.2	0.3	0.4	0.9
0.3	0.8	0.2	0.6	0.8	0.2
0.2	0.1	0.7	0.1	0.4	0.6

Find two or more numbers in a row—across or down— with a sum of 1.

10

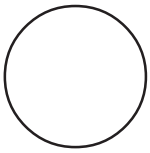
1.7	4.6	1.5	3.9	0.8	6.2
8.3	2.4	5.7	4.3	4.6	3.8
0.5	2.9	6.6	2.2	4.6	2.5
2.3	8.7	4.5	4.3	7.2	0.5
2.8	1.3	5.5	3.5	2.8	5.5
4.9	0.6	9.4	1.9	8.1	1.5

Find two or more numbers in a row—across or down— with a sum of 10.

5

4.3	0.7	1.6	2.5	2.5	1.6
2.3	2.0	3.4	4.2	1.9	3.4
2.7	3.0	2.1	0.8	3.1	1.1
1.8	5.0	2.9	3.7	1.3	3.9
3.2	0.0	4.7	1.5	1.7	4.0
1.0	4.0	0.3	3.5	3.3	1.0

Find two or more numbers in a row—across or down— with a sum of 5.



Find two or more numbers in a row—across or down— with a sum of _____.

Double number line

Instructions: Mark your number line as per instructions. Work with a partner to complete the instructions for each example.

- Record intervals by 500 from 0–5000 on your double number line. Take turns with your partner to write the following numbers on your line. Check your answer by flipping the strip.

2500 3750 375 1250 4050 2850 980

- Record intervals by 500 from 5000–10 000 on your double number line. Take turns with your partner to write the following numbers on your line. Check your answer by flipping the strip.

7500 8970 5500 6750 9900 8080 7050

- Record intervals by 1000 from 0–10 000 on your double number line. Take turns with your partner to write the following numbers on your line. Check your answer by flipping the strip.

2500 7500 5000 9250 3750 725 2375

- Record intervals by 1000 from 10 000–20 000 on your double number line. Take turns with your partner to write the following numbers on your line. Check your answer by flipping the strip.

15 000 11 025 17 650 10 900 13 050 18 700 14 750

Double number line

Counting On **framework reference: Level 4, Level 5**

This Activity is described on pages 60 to 63

This is an activity that can be given readily to a mixed ability class with students working with a partner of like ability. The adjacent worksheet can be used for students being extended to larger numbers, a proforma for this can be found on page 64. Students being extended to decimals can be given double number lines similar to those on page 63.

Note the extension idea suggested on page 60.

Make ten (adapted from and can be in conjunction with Make 100 pages 94 to 96)

Counting On **framework reference: Level 4**

Purpose

To help students add ones and tenths.

Materials needed

One die for the teacher (1–6 or 0–9); scoring sheets: one per student.

Time required

One to two lessons.

Rules

1. The aim is to score ten or as close to it as possible without "busting" (going past ten).
2. The teacher rolls the die and announces the number. The students may choose to divide the number by ten or score it at face value, e.g. 2 may be scored as 2 or 0.2.
3. The die is rolled again and the process repeated and the score recorded by the student at face value or one tenth of the face value. The student then records the progressive total in the adjacent column.
4. This continues until nine rolls have been completed. Note: All rolls must be used.
5. The student who scores ten or the number closest (but below) ten wins.

Variation

This can be played as a paired or small group activity with each student taking turns to roll the die for their own score.

The target can be changed to "Make one" with the die roll being divided by 10 or 100.

Allow subtraction of the die roll as a choice.

Sample game

Score	Total
0.2	.2
3	3.2
0.6	3.8
1	4.8
0.3	5.1
2	7.1
0.4	7.5
0.5	8.0
0.4	8.4
	8.4

Questioning

1. After two rolls the teacher asks students to name all possible scores. (In the sample game above the possible scores are 0.5, 2.3, 3.2, and 5.0.) Note: If the first two numbers rolled are the same, there are only three possible scores, e.g. rolls of 3, produce possible scores of 0.6, 3.3, 6.0.

2. When there are two rolls to go, ask students to stand if it is still possible for them to score exactly 10. Ask their scores and check as a whole class activity.

3. When there are two rolls to go, ask students to announce their scores and say what they would like the next two rolls to be, e.g. a student with a score of 8.8 could score exactly 10 with 2 sixes. The same result could be achieved with a 1 and a 2 (1.2).

Make 10

Score	Total

Score	Total

Score	Total

Score	Total

Score	Total

Score	Total

Decimal trisaw

Counting On **framework reference: Level 4**

Purpose

To observe student understanding of the relationship between common and decimal fractions.

Materials needed

One prepared square-saw per group. (To prepare a square-saw, photocopy the decimal square-saw onto light cardboard and cut into individual squares. It is a good idea to identify each set by using different colours or markings.

Blank tri-saw grid: one paper and one light cardboard version per group.

Time required

Two to three lessons.

Instructional sequence

1. Lead class discussion of different ways to represent the concept of *a half* before leading to class discussion of ways to represent simple decimal fractions as their equivalent fractions.
2. Have small groups reassemble the decimal square-saw and discuss the things that made the puzzle non-trivial, e.g. 0.25 appears with more than one matching edge.
3. Show how a tri-saw is constructed. Be careful to highlight the orientation of the writing.
4. Each group then brainstorms nine facts that could be placed onto a tri-saw. The challenge is to make the puzzle difficult to solve. The teacher needs to check the facts and may like to use this as a form of assessment of the level of understanding of the relationship between common and decimal fractions.
5. Have the group trial constructing the tri-saw on the paper version first. The teacher needs to check the orientation of the numbers and that there is sufficient space for cutting the edges.
6. Once checked, the student can make the cardboard version and give their puzzle to another group to solve.

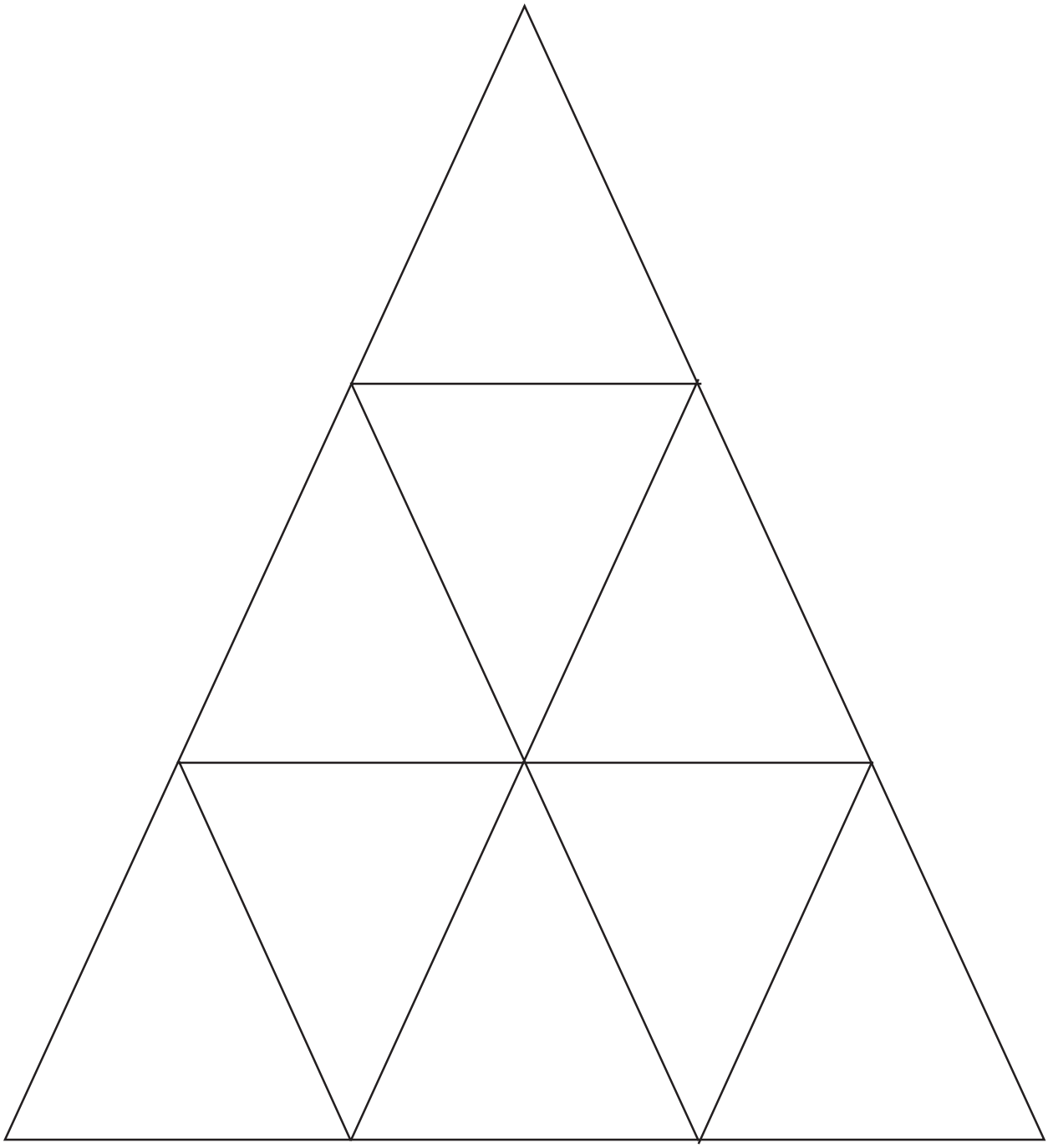
Variation

Have students include *distracters* for the edge of the puzzle.

Squaresaw

0.3 0.25	$\frac{3}{10}$ 0.4 $2\frac{1}{2}$	$\frac{2}{5}$ $\frac{70}{100}$ 0.5	0.7 0.9
$\frac{1}{4}$ $\frac{1}{10}$ 0.03	2.5 0.07 1.0 $\frac{75}{100}$	$\frac{1}{2}$ $\frac{4}{5}$ $\frac{7}{100}$ 0.25	$\frac{90}{100}$ 0.8 $\frac{1}{3}$
$\frac{3}{100}$ $\frac{7}{10}$ $\frac{1}{5}$	0.75 0.7 $1\frac{1}{2}$ $\frac{3}{4}$	$\frac{25}{100}$ 0.5 7.5 0.9	$0.3\dot{3}$ $\frac{50}{100}$ $\frac{19}{100}$
0.2 1.9	0.75 $1\frac{1}{10}$ $\frac{30}{100}$ $\frac{3}{4}$	$\frac{9}{10}$ 0.6 0.3	0.19 $\frac{3}{5}$

Trisaw



Decimal sorting

Counting On **framework reference: Level 4**

Purpose

To extend understanding of numbers between zero and one in relation to each other.

Materials needed

Prepare one set of cards for each pair of students.

Time required

One lesson.

Instructional sequence

In pairs, students sort cards into three groups: Near 0, About 1/2 and near 1. Students should explain why they have placed the cards in particular groups.

Extension

Students could order the cards.

Students could make up two more cards for each group.

0.1	0.53	0.90
0.51	0.2	0.48
0.89	0.55	0.95
0.09	0.12	1.06
Near 0	Near 1/2	Near 1

Where's that number?

Counting On **framework reference: Level 4**

Purpose

This activity builds on from the *Ordering whole numbers to 100* (page 58) and *double number line* (page 60) activities.

Materials needed

Photocopied worksheets: one per student.

Time required

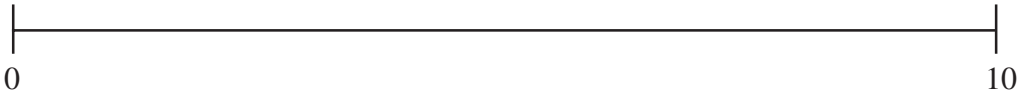
One lesson.

Instructional sequence

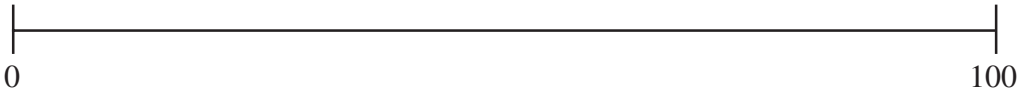
These worksheets may be introduced by using the clothesline activity with the end points of the clothesline chosen to match the level of the worksheet.

Worksheet 1

Place these numbers on the line



7, 9

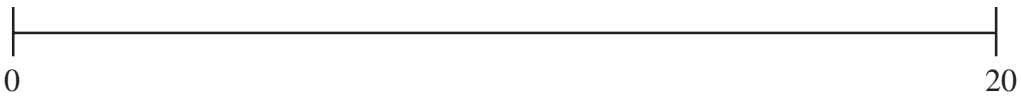


20, 75

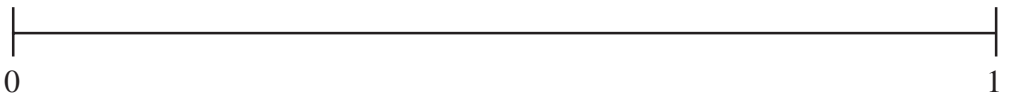
15, 3

6, 8

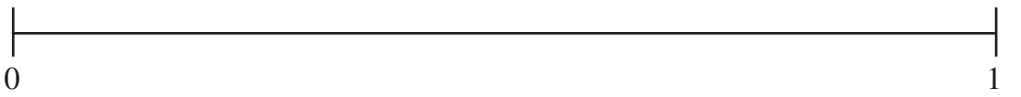
70, 95



2, 12



0.3. 0.6

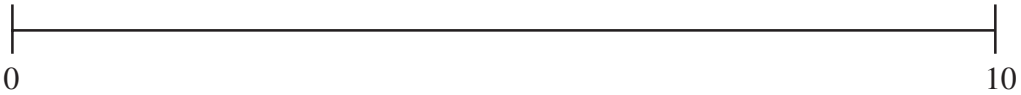


0.9, 0.4

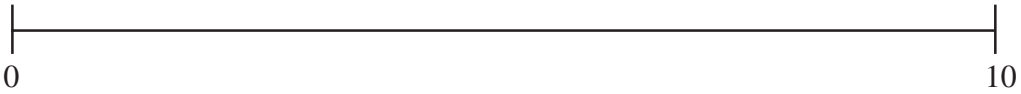
Hint: Mark and label the middle of each line before you start.

Worksheet 2

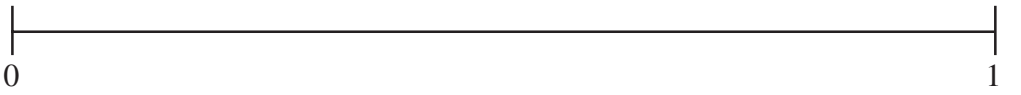
Place these numbers on the line



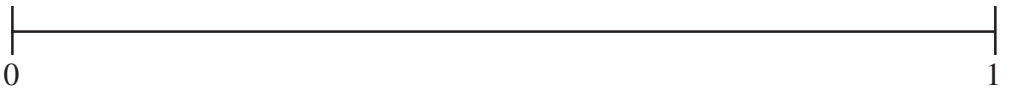
8, 3



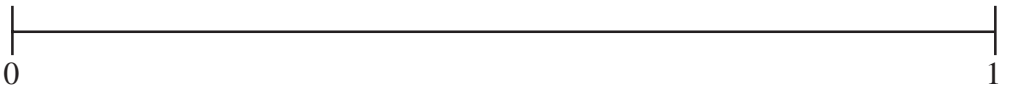
1, 7



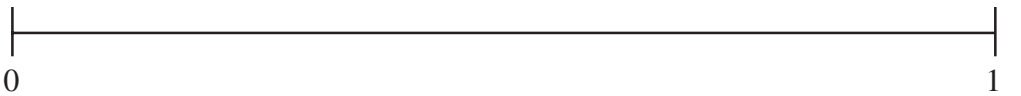
0.6, 0.3



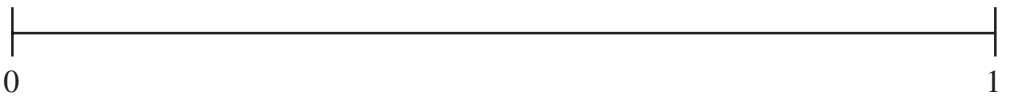
0.2, 0.09



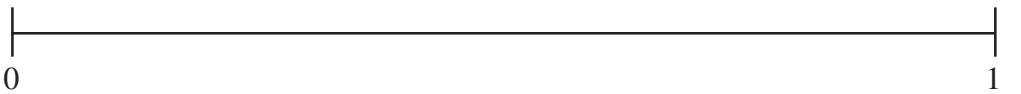
0.75, 0.7



0.3, 0.15



0.99, 0.10

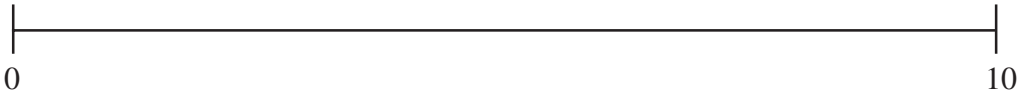


0.25, 0.4

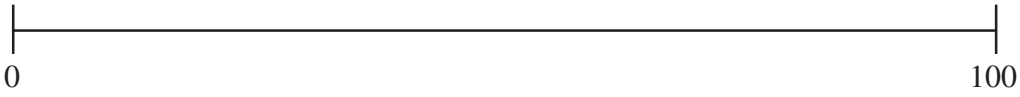
Hint: Mark and label the middle of each line before you start.

Worksheet 3

Place these numbers on the line

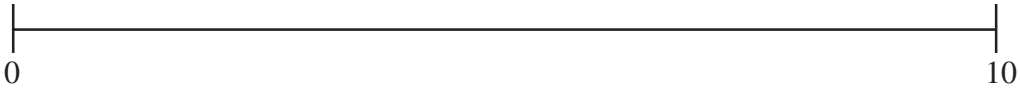


3, 5.5

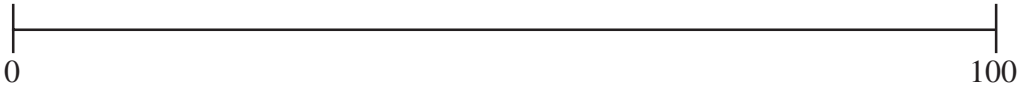


0.3, 0.45

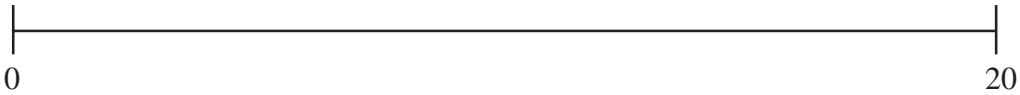
0.8, 0.35



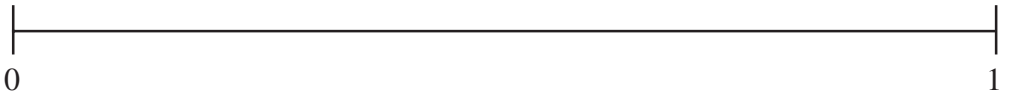
0.6, 0.08



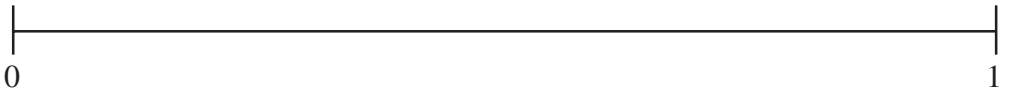
0.1, 0.15



0.02, 0.07



0.08, 0.03



0.06, 0.025

Hint: Mark and label the middle of each line before you start.

Decimal jigsaw

Counting On **framework reference: Level 4**

Purpose

To strengthen understanding of the patterns inherent in our base ten system.

Materials needed

- Prepare sufficient jigsaws. This activity is usually done in pairs but can be done individually.
- 1 OHT of a hundred grid or a hundred chart, p. 82.
- 1 OHT of a zero to ten grid, p. 209.

Time required

One lesson.

Instructional sequence

- Display the hundred grid or chart and discuss patterns when moving down, up or diagonally around the chart.
- Display the OHT of the zero to ten chart and draw out similarities.
- Distribute the jigsaws, explaining that the students are expected to reassemble the pieces to form the grid.

Variation

Copy the jigsaw onto paper and cut it up. Shuffle the pieces around and distribute onto one or two pieces of A4 paper and re-photocopy for the students. Have the students write in the missing numbers.

Extension

Use the zero to one chart, p. 211.

Grid for 0.1 to 10 with missing numbers

		0.3						0.9	
1.1					1.6				2.0
						2.7			
			3.4				3.8		
4.1					4.6				5.0
	5.2			5.5				5.9	
		6.3					6.8		
				7.5		7.7			
	8.2		8.4						
9.1				9.5				9.9	

NS3.4

Preparation of jigsaw

Copy the grid onto light card and cover with *contact* or laminate if possible.

Use a different colour card for each puzzle, or rule over grid lines with a highlighter, so lost pieces can be returned to the correct puzzle.

Cut the grid into jigsaw pieces, along the **dark** lines. Store jigsaw pieces in a sandwich bag or envelope.

How to play

Students put jigsaw pieces together to make the complete grid.

Design your own jigsaw

0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.2
0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.3
0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.4
0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.5
0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59	0.6
0.61	0.62	0.63	0.64	0.65	0.66	0.67	0.68	0.69	0.7
0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.78	0.79	0.8
0.81	0.82	0.83	0.84	0.85	0.86	0.87	0.88	0.89	0.9
0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1

NS3.4

Instructions

Each student has a copy of the grid.

Students;

- use a felt pen to highlight the cutting edges
- white out all numbers except for one number per jigsaw piece
- cut out jigsaw for other students to reassemble

Decimal snakes and ladders

Counting On **framework reference: Level 4**

Purpose

To consolidate understanding of tenths and hundredths.

Materials needed

Place students in small groups with each student having a counter and each group having a game board and a die.

Time required

One lesson.

Instructional sequence

This game is played in the same way as normal snakes and ladders except that the students are expected to state the number they will land on before moving the counter.

Variation

Start at one and move backwards to zero. The rules may be varied to go down the ladders and up the snakes.

Decimal snakes and ladders

FINISH

Go ahead 0.05 0.91	0.92	0.93	0.94	0.95	0.96	0.97	Go back 1 tenth 0.98	0.99	1.00
0.90	0.89	0.88	0.87	0.86	Miss a turn 0.85	0.84	0.83	0.82	0.81
Go back 2 tenths 0.71	0.72	0.73	Go forward 7 hundredths 0.74	0.75	0.76	Have another turn 0.77	0.78	0.79	Go forward 6 hundredths 0.80
0.70	0.69	0.68	0.67	0.66	0.65	Go ahead 0.05 0.64	0.63	0.62	0.61
0.51	0.52	Go back 1 tenth 0.53	0.54	0.55	0.56	0.57	0.58	Go ahead 0.10 0.59	0.60
0.50	0.49	0.48	0.47	0.46	0.45	0.44	0.43	Go forward 1 tenth 0.42	0.41
0.31	0.32	0.33	Miss a turn 0.34	0.35	0.36	0.37	0.38	0.39	0.40
Have another turn 0.30	0.29	0.28	0.27	Go ahead 0.10 0.26	0.25	Go forward 4 hundredths 0.24	0.23	0.22	0.21
0.11	0.12	0.13	Go back 1 tenth 0.14	0.15	0.16	0.17	Go ahead 0.05 0.18	0.19	0.20
Go back 1 hundredth 0.10	0.09	0.08	0.07	0.06	Move forward 3 hundredths 0.05	0.04	Miss a turn 0.03	0.02	0.01

START

Introducing percentages and decimals using a number ribbon

Counting On **framework reference: Level 4**

Percentages and decimals

Rationale

The primary mathematics syllabus emphasises the importance of decimal fractions as the most commonly observed form of fractions. The teaching units in support of the syllabus, *Fractions and decimals 4–7*, introduce hundredths before tenths. The rationale for emphasising decimals to two decimal places and starting with hundredths was the ready availability of money as a common example of recording to two decimal places. In addition, the Multibase Arithmetic Blocks (MAB) used as a model of place value in the syllabus could be used to produce an area model to support decimals. This was achieved by changing the value of a "flat" from 100 to 1.

With the removal of one-cent and two-cent coins, money no longer adequately models decimal fractions to two decimal places. Further, the work of Carraher (1988) in the study of "street mathematics in Brazil" indicated that some Brazilian street children could perform mathematics when making sales in the street but were unable to answer similar problems presented in school. It is possible that for many students, the use of money that we take to indicate an appreciation of a decimal system is actually a form of barter — so many of these for this number of these.

Beginning with a linear model of percentages, as suggested by Moss and Case (1999), allows children to take advantage of their knowledge of the numbers 1 to 100 as well as focusing on continuous quantities and measurement.

Students should have had experience with locating numbers on the clothesline model using 0 at one end and 100 at the other before introducing the next sequence of activities.

Download

Purpose

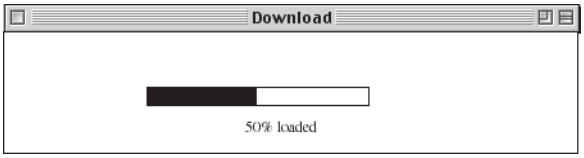
To develop a context for a linear model of percentages that builds upon students' understanding of 100 as a unit.

1. Whole class.

   Introduce the context:

   Sometimes when you watch a computer program being loaded you might see a little bar that is progressively coloured in. Has anyone ever seen this?

   I am going to draw a download bar on the board and I am going to colour it in using the side of the chalk. I want you to put up your hand when you think that it shows 50% loaded.



Gain consensus as to where it will show 50% loaded. Leave 50% shaded on the board. Select a student to come to the board and to move a vertical marker (such as a pencil or rule) in front of the download bar.

When I tell you to, I want you to hold your rule up straight and move it slowly across in front of the bar to show how much has been loaded. This time I want you all to put up your hands when you think that 25% has been loaded.

Have the student at the board stop when the first hand goes up.

Do you think that 25% has been loaded? Why do you think that this is 25%?

Repeat the process for 75%.

Erase the 50% loaded mark and invite a student to the board to show where he or she believes 33% loaded would be. Ask the student to justify his or her answer.

2. Small group work

   Using a 12 cm strip of coloured tape or paper, show 50%, 25% and 75% by folding.

   Using a new strip of paper, can you show where 20% loaded would be?

   Now can you show 40%, 60%, and 80%?

   Can you show 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90% and 100% by folding, cutting and pasting?

   Using 12 cm rectangles the same width as the tape on a worksheet, have students record the downloads.

   Use the results to estimate 15%, 35%, 45%, 55%, 65% and 95%.

   Using a 12 cm strip of coloured tape or paper to represent 100%, have your students show 125%.

Extension:

You can use a number line marked on the playground with one metre markings to support the transition from percentages to decimals. For example, "When you pass the two metre mark and walk 75% of the way to the three metre mark, the point you reach can be written as 2.75 m."

Notes

Unit 2: Multiplication and division

Using Multo to assess student understanding

(extension from page 177)

Counting On **framework reference: Level 5 multiplication and division**

Purpose

To provide an opportunity for observational assessment, and a student work sample for the assessment of student understanding in multiplication facts and factors.

Materials needed

Multo 1 and Multo 2 worksheets for each student.

Two sets of 1 to 10 cards or two 0 to 9 dice with the zero marked as 10 or a set of 100 cards with the multiplication facts from 1 x 1 to 10 x 10.

Time required

One lesson

Instructional sequence

- If students have not played Multo before, play one game. (See page 177)
- Distribute the Multo 1 grids and lead a class discussion on the first grid. Highlight numbers which could not be attained, e.g. 46
- Set students to work individually to complete the worksheet.
- Collect the sheets and discuss strategies for producing the ideal Multo grid.
- Hand out Multo 2 for completion.
- Observe the numbers students are using. Hint at "good" numbers as the ones with several factors.
- Once all students have filled their grid, play the game a few times.

Variation

When playing the final series of games allow students to make some alterations to their numbers provided they record what they changed and why.

Multo 1

Student name: _____ Date: _____

NS3.3 Selects and applies appropriate strategies for multiplication and division

WMS3.2 Selects and applies appropriate problem-solving strategies including technological application in understanding

WMS3.4 Gives a valid reason for supporting one possible solution over another

WMS3.3 Describes and represents a mathematical situation in a variety of ways using mathematical terminology and some conventions

Julie, Michael and Nathan made some game boards. Explain what you think each student's chances of winning might be. Give reasons for your answer.

Julie

12	5	29	36
28	46	87	50
81	54	14	8
63	10	7	35

Michael

1	2	3	4
20	44	90	79
18	25	9	10
37	36	35	34

Nathan

16	9	18	24
5	21	6	30
14	48	72	45
12	33	17	20

_____

_____

_____

_____

_____

_____

_____

_____

Comment: _____

_____

Multo 2

Student name: _____ Date: _____

NS3.3 Selects and applies appropriate strategies for multiplication and division

WMS3.2 Selects and applies appropriate problem-solving strategies including technological application in understanding

WMS3.4 Gives a valid reason for supporting one possible solution over another

WMS3.3 Describes and represents a mathematical situation in a variety of ways using mathematical terminology and some conventions

Fill in this Multo grid. You may only use each number once. Explain why you chose those numbers.

_____

_____

_____

_____

_____

_____

_____

_____

Comment: _____

_____

Beat the calculator

Counting On **framework reference: Place value, Level 4; Multiplication and division Level 5**

Purpose

Discovery of the *rule* for multiplication of a decimal by a decimal. This activity could be used as a catalyst for discussion as described in the next activity.

Materials needed

- Prepare an OHT of the worksheet plus one paper copy per student.
- One calculator per student or shared between two students.

Time required

One lesson

Instructional sequence

- Display OHT with the first row of questions visible.
- Have students use their calculators to find the four solutions and record the results on the OHT.
- Discuss "What did you notice?"
- Hand out the worksheet and have students predict the answers to the next four questions and then check them on their calculator.
- Discuss the *rule* or pattern that they have observed.
- In pairs play *Beat the calculator*:
  - Both students record the answer to the question 42 x 15.
  - One student agrees to use the calculator for the remaining questions while their partner agrees to work without the calculator.
  - Begin the race with "Go!"
- Have students write in their own words what they discovered by doing this activity.

Multiplication of decimals

Name: _____

What can you notice about the placement and movement of the decimal point?

$$\begin{array}{r} 38 \\ \times\ 12 \\ \hline \end{array} \qquad \begin{array}{r} 38 \\ \times\ 1.2 \\ \hline \end{array} \qquad \begin{array}{r} 38 \\ \times\ 0.12 \\ \hline \end{array} \qquad \begin{array}{r} 3.8 \\ \times\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 0.38 \\ \times\ 12 \\ \hline \end{array} \qquad \begin{array}{r} 3.8 \\ \times\ 1.2 \\ \hline \end{array} \qquad \begin{array}{r} 0.38 \\ \times\ 0.12 \\ \hline \end{array} \qquad \begin{array}{r} 0.38 \\ \times\ 0.012 \\ \hline \end{array}$$

Can you beat the calculator?

$$\begin{array}{r} 42 \\ \times\ 15 \\ \hline \end{array} \qquad \begin{array}{r} 42 \\ \times\ 1.5 \\ \hline \end{array} \qquad \begin{array}{r} 42 \\ \times\ 0.15 \\ \hline \end{array} \qquad \begin{array}{r} 4.2 \\ \times\ 15 \\ \hline \end{array}$$

$$\begin{array}{r} 0.42 \\ \times\ 15 \\ \hline \end{array} \qquad \begin{array}{r} 4.2 \\ \times\ 1.5 \\ \hline \end{array} \qquad \begin{array}{r} 0.42 \\ \times\ 0.15 \\ \hline \end{array} \qquad \begin{array}{r} 0.42 \\ \times\ 0.015 \\ \hline \end{array}$$

Investigating decimal multiplication

Counting On **framework reference: Place value, Level 4; Multiplication and division Level 5**

Purpose

To have students analyse the meaning of multiplying a decimal by a decimal.

Time required

Part of one lesson for the introduction and one later lesson for student reporting.

Task

Students are to investigate the meaning of 0.1 x 0.1 and present their findings in written and verbal form.

Instructional sequence

- Remind students of the pattern they discovered during the *Beat the calculator* lesson.
- Review the relationship between fractions and decimals including in the review 0.1 and one tenth.
- Use a Think—Pair—Share strategy to elicit the student understanding of multiplication.
- Ask the students to think about the meaning of 0.1 x 0.1 and prepare an explanation, which is not to be more than one page, and which can include diagrams. Inform them that a random selection or students will be asked to present their findings to the whole class.
- At the later lesson, collect all presentations and use a random way of selecting students to present their findings.
- Use the student presentation to lead a class discussion. Relate what they have found with the *Beat the calculator* lesson.
- Have the class negotiate the "Rule for Multiplying Decimals" and make a poster of the rule for the classroom wall.

Hex

Counting On **framework reference: Place value, Level 4; Multiplication and division Level 5**

Purpose

This activity is designed to have students predict the answer to a multiplication or division question and verify their answer on the calculator.

Materials needed

- One game board between two teams. A team can be one or two students.
- Coloured counters with each team on a game having their own identifiable colour,
- One calculator per game.

Time required

One lesson.

Instructional sequence

- Teams take turns. Pick any two numbers in the cloud, stating the two they have chosen.
- Divide the numbers you picked. e.g. 20 ÷ 75 or 75 ÷ 20
- If the answer is on the game board, place your teams counter on it.
- The first team to get a path of answers connecting its two sides of the game board wins.

Variation

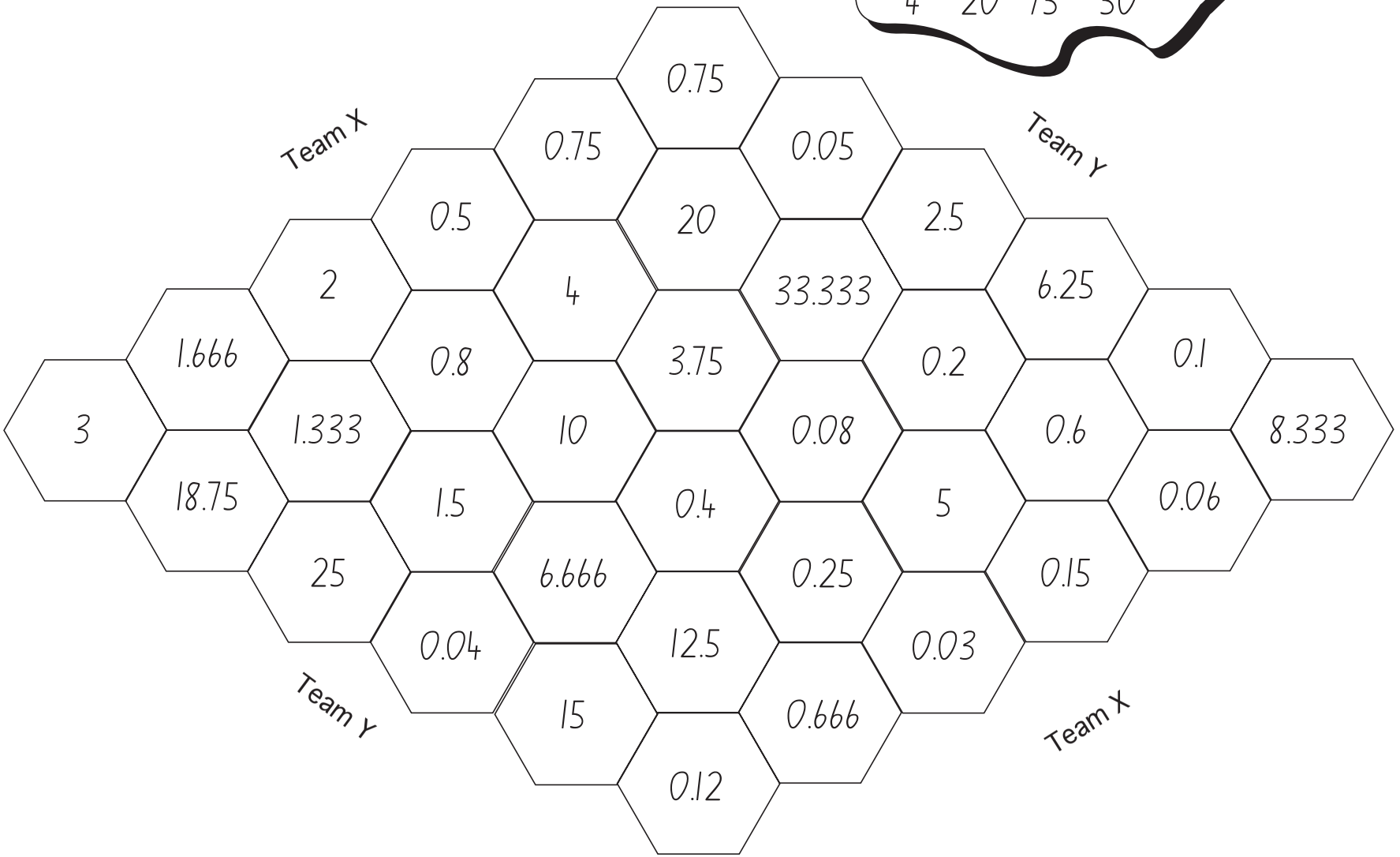
When the team predicts their answer the other team checks this on the calculator. If the prediction is incorrect the second team gets to place their counter on the correct answer.

Extension

Decimal Hex which allows multiplication or division.

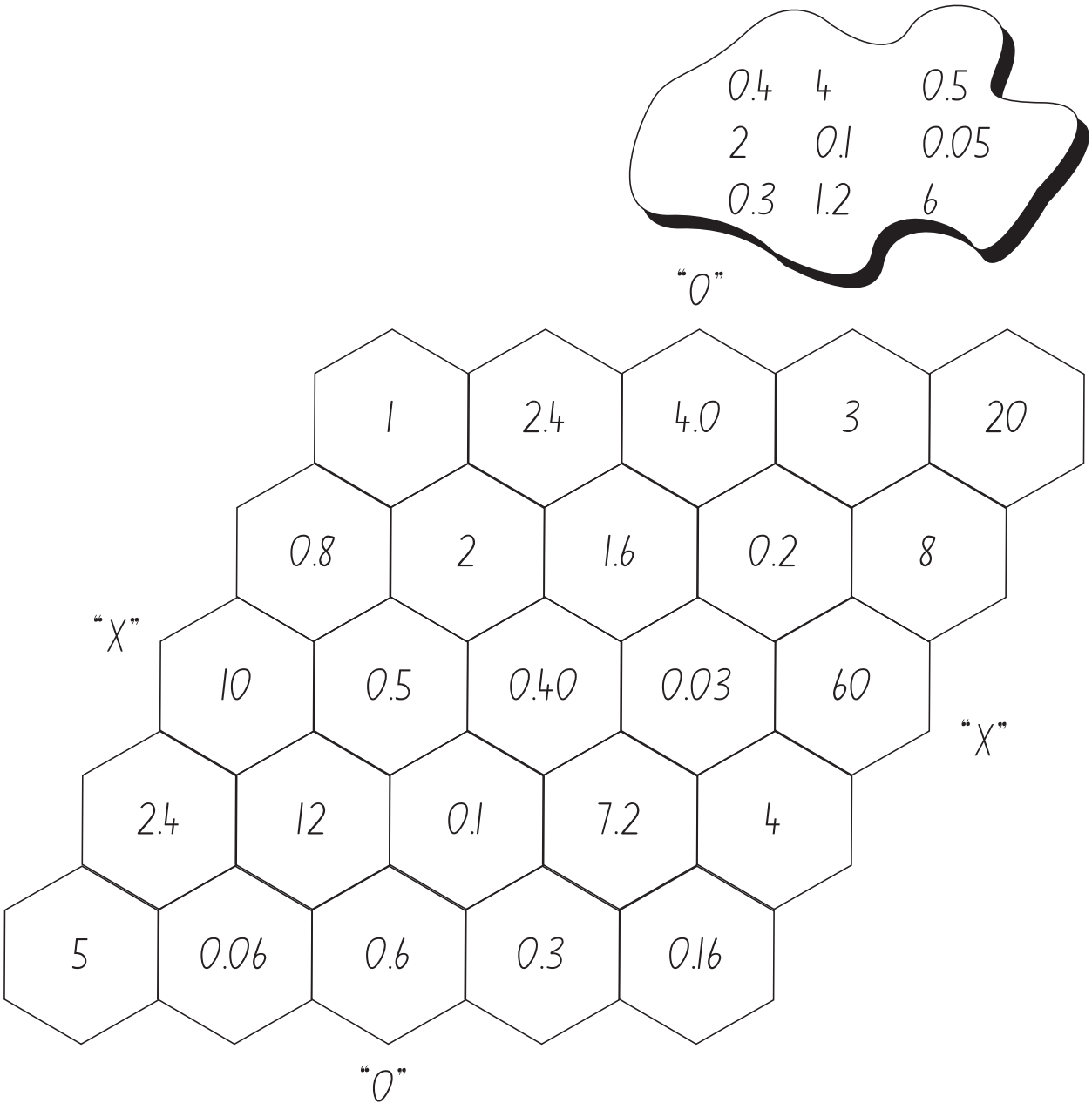
Hex game

3 5 25 100
4 20 75 50



Decimal hex

Using only these numbers, choose any two, and multiply or divide them to find the hexagon which contains the answer.



Unit 3: Patterns and algebra

AFLgebra

PAS4.2 Creates, records, analyses and generalises number patterns using words and algebraic symbols in a variety of ways.

Counting On framework—no direct link.

Purpose

To connect work done within the Counting On framework to a known situation and linking to an introduction to algebra.

Materials needed

Soft football, scoring sheets and dice for student game.

Instructional sequence

1. On the board, draw the AFLgebra goalposts as shown:

Miss | Behind | Goal | Behind | Miss

2. Select a player for each team.
3. Wearing a blindfold, the first player throws the ball at the board. Scoring is as follows:
   - A "goal" is worth six points
   - A "behind" is worth one point
   - A "miss" scores no points.
4. Scoring is done on a hundreds chart by circling the cumulative total after each scoring shot. e.g. If, after ten shots, a player had scored a goal, a behind, a goal and seven misses the scoring chart would read:

G	B	G	M	M	M	M	M	M	M

1	2	3	4	5	(6)	(7)	8	9	10
11	12	(13)	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

$2\square + 1\triangle = (13)$

5. Players have four alternate "quarters" made up of either ten throws, or as many throws as they can in one minute.
6. The winning team is the team that scores the most points when all quarters are added up.

Variation

Have students play the game in pairs using dice rolls to score. Rolling a 1, 2, or 3 scores a "behind" and a 4, 5 or 6 scores a "goal".

Extension

Write the results for one game on the board, e.g.

$2\square + 1\triangle = 13$

and add some examples with one coefficient missing, e.g.

$\square + 3\triangle = 21.$

Extend this to an example with both coefficients missing, e.g.

$\square + \triangle = 19$

could be $3\square + 1\triangle = 19$

or $2\square + 7\triangle = 19$

and collect a variety of responses.

Now use another scoring system from other sports, e.g.

(Rugby league) $3\square + 4\triangle = 20$

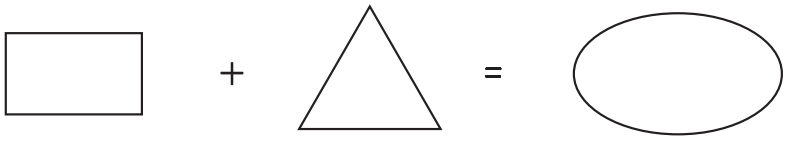
(Basketball) $3\square + 4\triangle = 17$

and challenge students to "name the game".

AFLgebra scorecard 1st quarter

1	2	3	4	5	6	7	8	9	10

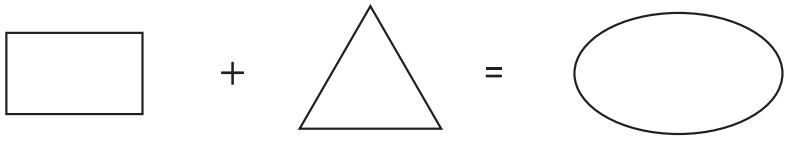
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



AFLgebra scorecard 2nd quarter

1	2	3	4	5	6	7	8	9	10

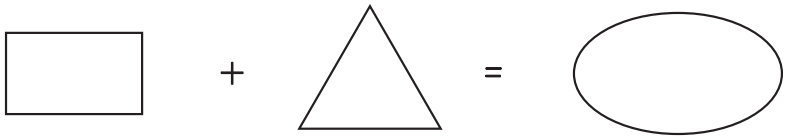
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



AFLgebra scorecard 3rd quarter

1	2	3	4	5	6	7	8	9	10

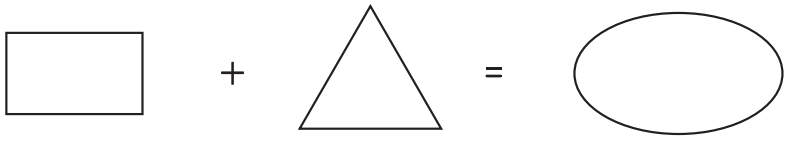
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



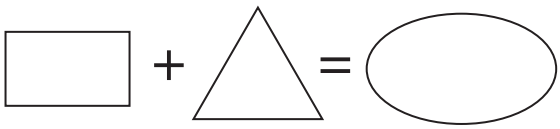
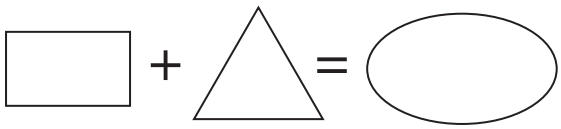
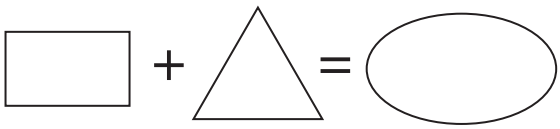
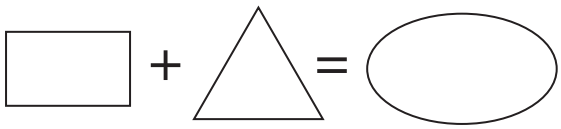
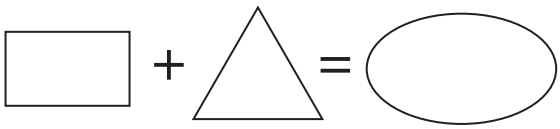
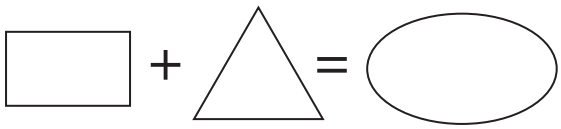
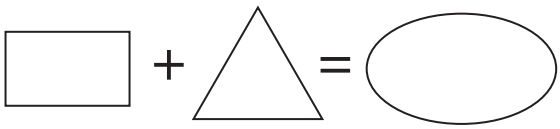
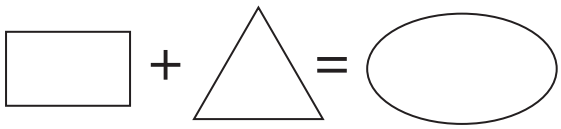
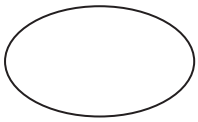
AFLgebra scorecard 4th quarter

1	2	3	4	5	6	7	8	9	10

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



AFLgebra tally sheet

Team A	Team B
	
	
	
	
Total = 	Total =